PARAMETRIC FAST FOURIER TRANSFORM AND WAVELET EXPANSIONS

Vasily N. Malozemov[†], Oleg V. Prosekov[‡]
[†]St. Petersburg State University, Russia
[‡]ARC International, St. Petersburg, Russia
[†]*malv@math.spbu.ru*, [‡]*Oleg.Prosekov@ARC.com*

The general approach to constructing FFTs involves the decomposition of the Fourier matrix into a product of sparse matrices. Various versions of this decomposition depend on the arithmetic properties of the order of the Fourier matrix and on representations of its indices. In this talk we present the parametric coding of indices and obtain the “perfect” parametric decomposition of the Fourier matrix.

Let $N = n_1 n_2 \cdots n_s$, $\Delta_\nu = n_1 n_2 \cdots n_{\nu-1}$ (for $\nu \in 2 : s+1$, $\Delta_1 := 1$) and $N_\nu = n_{\nu+1} n_{\nu+2} \cdots n_s$ (for $\nu \in 0 : s-1$, $N_s := 1$).

Theorem. For any parameter vector $p = (p_1, p_2, \ldots, p_s)$, the Fourier matrix F_N admits the representation

$$F_N = \left(Rev^{(q_1,\ldots,q_s)}_{n_1,\ldots,n_s}\right)^T \left(\prod_{\nu=1}^{s} \left(I_{N_\nu} \otimes Twid^{(p_1,\ldots,p_{\nu-1},q_\nu)}_{n_1,\ldots,n_{\nu-1},n_\nu}\right) \times \right.$$
$$\left. \times \left(I_{N_\nu} \otimes F_{n_\nu} \otimes I_{\Delta_\nu}\right) \right) Mix^{(p_1,\ldots,p_s)}_{n_1,\ldots,n_s},$$

where $\otimes$ denotes Kronecker multiplication of matrices, I_N is the identity matrix of order N and $q = (q_1, q_2, \ldots, q_s)$ is the adjoint parameter vector with elements are defined by the condition $\langle q_\nu \, p_\nu \rangle_{n_\nu} = 1$ for $\nu \in 1 : s$.

Also this decomposition involves special matrices: permutation matrices *Rev*, *Mix* and diagonal matrix *Twid*. The parameter vector can be chosen so that the number of nontrivial elements (different from ± 1 and $\pm i$) in the parametric twiddle matrix decreases in comparison with the usual (nonparametric) twiddle matrix. Incidentally, permutation matrices have its decompositions.

Basing on this factorization, we construct sequences of orthogonal bases in a signal space which generate a parametric wavelet packet.

References

1. Malozemov V. N., Prosekov O. V. *Parametric Versions of the Fast Fourier Transform* // Doklady Mathematics. 2008. Vol. 78. No. 1. P. 576–578.