TIGHT FRAMES OF SPECIAL FORM

Natalia A. Solovjova
Saint-Petersburg State University,
The Faculty of Mathematics and Mechanics,
Universitetsky prospekt, 28, 198504, Peterhof,
St. Petersburg, Russia.
4vinyo@gmail.com

Suppose U is a unitary $(n \times n)$–matrix with eigenvalues $\lambda_1, \dots, \lambda_n$ and corresponding orthonormal eigenvectors $p_1, \dots, p_n$. Let us fix a unit-norm vector $\varphi_0 \in \mathbb{C}^n$ and assuming that $m \geq n$ build the vector system

$$\left\{\varphi_0, U\varphi_0, U^2\varphi_0, \dots, U^{m-1}\varphi_0\right\}. \tag{1}$$

THEOREM 1. *The system* (1) *is a tight frame if and only if two following conditions hold:*

(α) *the eigenvalues $\lambda_1, \dots, \lambda_n$ of the matrix U are distinct m-roots of a certain number $c \in \mathbb{C}$, $|c| = 1$;*

(β) $|\langle \varphi_0, p_k \rangle| = \frac{1}{\sqrt{n}}$ *for all $k \in 1 : n$.*

Let us examine the vectors

$$\eta_k = P^* \varphi_k, \qquad k \in 0 : m-1,$$

where P is a unitary matrix with the columns $p_1, \dots, p_n$.

THEOREM 2. *The system $\{\eta_0, \eta_1, \dots, \eta_{m-1}\}$ is a general harmonic frame if and only if conditions* (α) *and* (β) *of the theorem* 1 *hold.*

As an application of theorem 2 it is shown how to reduce the Mercedes-Benz frame to a general harmonic frame.

References

[1] P. G. Casazza and J. Kovačević, «Equal-norm tight frames with erasures», *Adv. Comp. Math.* (*Special Issue Frames*), vol. 18, no. 2–4, pp. 387–430, 2002.