Министерство образования и науки РФ
Санкт-Петербургское отделение математического института
им. В.А. Стеклова РАН
Воронежский государственный университет

International Conference
WAVELETS AND APPLICATIONS

June 14-20. 2009, St.Petersburg, Russia

ABSTRACTS

Санкт-Петербург 2009

УДК 517.5

Международная конференция "**Всплески и приложения**, 14-20 июня 2009 г. Санкт-Петербург, Россия". Тезисы докладов

ОРГКОМИТЕТ

Издание осуществлено при поддержке Российского фонда фундаментальных исследований (проект № **09-01-06036-г**)

Компьютерная верстка и подготовка оригинал-макета: **Северов П.Г.**

CONTINUOUS WAVELET TRANSFORM ON THE ONE-SHEETED HYPERBOLOID

A. Askari Hemmat, Z. Yazdani Fard
Department of Mathematics, Shahid Bahonar University of Kerman, Kerman, Iran
askari@mail.uk.ac.ir

The one-sheeted hyperboloid $H^{1,1}$ with Cartesian equation $x_0^2 - x_1^2 - x_2^2 = -1$ may be parametrized as $x = (\sinh\chi,\ \cosh\chi\cos\varphi,$ $\cosh\chi\sin\varphi)$, where $\chi \in \mathcal{R},\ \ 0 \leq \varphi < 2\pi$. The motions on $H^{1,1}$ are of two types: (i) rotations: $x(\chi,\ \varphi) \mapsto (\chi,\ \varphi + \varphi_0)$; and (ii) displacements: $x(\chi,\ \varphi) \mapsto (\chi + \chi_0,\ \varphi)$. They constitute the group $SO_0(1,\ 2)$. To define dilation on $H^{1,1}$, project it onto the cone $\mathcal{C} = \{\xi = (\xi_0,\ \xi_1,\ \xi_2) \in \mathbb{R}^3 : \xi_0^2 - \xi_1^2 - \xi_2^2 = 0\}$, with dilatin $\xi \mapsto a\xi$. In polar coordinates, the dilation oprator acts on a point $x(\chi,\ \varphi)$ by $D_a(\chi,\ \varphi) = (\chi_a,\ \varphi)$, with $sinh\chi_a = asinh\chi$.
For all test functions f on $H^{1,1}$ introduce the following pair of transforms $\hat{f}_\pm(\nu,\ \xi) = \langle f(x),\ \varepsilon^\pm_{\nu,\ \xi}(x)\rangle$, where $\nu \in \mathbb{R}^+, \xi$ varies on the half cone $\mathcal{C}^+ = \{\xi \in \mathcal{C} : \xi_0 > 0\}$ and the kernels $\varepsilon^\pm_{\nu,\ \xi}(x)$ are *hyperbolic plane waves*. This transformation is called the *Fourier-Helgason transform.*
Let $\psi \in L^2(H^{1,1})$ be a symmetric and rotation invariant function, $a \mapsto \alpha(a)$ a positive function on $\mathbb{R}^+_*$, we say that ψ is *admissible* if there exists constants m and M such that$0 < m \leq \mathcal{A}_\psi(\nu) := \int_0^\infty |(\widehat{\psi_a})_\pm(\nu)|^2 \alpha(a) da \leq M < \infty$. Given an admissible $H^{1,1}$-wavelet ψ, the $H^{1,1}$-*continuous wavelet transform* of $f \in L^2(H^{1,1})$ is $W_f(a,\ g) := \langle\psi_{a,\ g},\ f\rangle = \int_{H^{1,1}} \overline{\psi_a(g^{-1}x)} f(x) d\mu(x),\ g \in SO_0(1,\ 2),\ \ a > 0$ where $\psi_a(x) = \lambda(a,\ x)^{\frac{1}{2}} \psi(D_{\frac{1}{a}} x)$ such that $\lambda(a,\ x)$ is the Radon-Nikodym derivative. We show that a symmetric and rotation invariant function $\psi \in L^2(H^{1,1})$ is admissible if $\alpha(a)da$ be a homogeneous maesure of the form $\alpha^{-\beta}da$ with $\beta > 3$ and the following zero-mean condition is satisfied $\int_{H^{1,1}} \psi(\chi,\ \varphi) d\mu(\chi,\ \varphi) = 0$.

References

1. S.T. Ali, J-P. Antoine, and J-P. Gazeau, Coherent States, Wavelets and Their Generalizations, Springer-Verlag, New York, Berlin,

Heidelberg, 2000.

2. J. Bros and U. Moschella, Fourier analysis and holomorphic decomposition on the one-sheeted hyperboloid, (2003) [arXiv: math-ph/0311052v1].

DECONVOLUTION BY MATCHING PURSUIT USING SPLINE WAVELET PACKETS DICTIONARIES

Amir Z. Averbuch, Valery A. Zheludev
School of Computer Science, Tel Aviv University, Israel
zhel@post.tau.ac.il

We present an efficient method that restores signals from strongly noised blurred discrete data. The method can be characterized as a regularized matching pursuit (MP), where dictionaries consist of spline wavelet packets. It combines ideas from spline theory, wavelet analysis, theory of ill-posed problems and greedy algorithms. The computational engine, which enables to construct versatile libraries of spline wavelet packet dictionaries and fast implementation of the algorithm, is the Spline Harmonic Analysis (SHA). SHA imposes harmonic analysis methodology onto spline spaces. It is especially applicable to convolution operations. The use of splines enables to map the discrete noised data into spaces of continuous functions, which approximate the sought after solution in the proper smoothed class. The main distinction from the conventional MP is that the different dictionaries are used to test data and to approximate the solution. In addition, the regularized correlation coefficients instead of the conventional ones are used. The regularizing parameters and the stopping rule for the algorithm are determined automatically. Experimental results exhibit a highly efficient algorithm. The coherent structure of the signals, which were subjected to the strong blurring and immersed into deep noise, were extracted.

AN HYBRID ALGORITHM FOR DATA COMPRESSION

Amir Z. Averbuch[1], Valery A. Zheludev[1], Moshe Guttmann[1], Dan D. Kosloff[2]
[1]School of Computer Science
Tel Aviv University, Israel
[2]Department of Earth and Planetary Sciences
Tel Aviv University, Israel
amir@math.tau.ac.il

We present an algorithm that compresses two-dimensional data arrays, which are piece-wise smooth in one direction and have oscillating events in the other direction. Seismic, hyper-spectral and fingerprints data have this mixed structure. The transform part of the compression process is an algorithm that combines wavelet and the local cosine transform (LCT). The quantization and the entropy coding parts of the compression were taken from the SPIHT codec. To efficiently apply the SPIHT codec to a mixed coefficients array, reordering of the LCT coefficients takes place. This algorithm outperforms other algorithms that are based only on the 2D wavelets transforms. Its compression capabilities are also demonstrated on multimedia images that have a fine texture. The wavelet part in the mixed transform of the hybrid algorithm utilizes the library of Butterworth wavelet transforms.

TO ONE ADDITIONAL EXTREMAL PROPERTY OF REGULAR SIMPLEX

V.F. Babenko[1], Yu.V. Babenko[2], N.V. Parfinovych[1], D.S. Skorokhodov[1]
[1] Dnepropetrovsk National University, UKRAINE,
[2] Sam Houston State University, Huntsville, TX, USA
babenko.vladislav@gmail.com, YVB001@shsu.edu, nparfinovich@yandex.ru, dmitriy.skorokhodov@gmail.com

For a d-dimensional simplex $\mathcal{T}$, let $L_p(\mathcal{T})$, $1 \le p \le \infty$, be the space of functions $f : \mathcal{T} \to \mathbb{R}$ endowed with the usual norm $\|\cdot\|_{L_p(\mathcal{T})}$. Denote $g_\pm(\mathbf{x}) = \max\{g(\mathbf{x}); 0\}$, $\mathbf{x} \in \mathbb{R}^d$. Let $\alpha, \beta > 0$ be given. For $f \in L_p(\mathcal{T})$, the asymmetric L_p-norm is defined as follows

$$\|f\|_{L_{p;\alpha,\beta}(\mathcal{T})} := \|\alpha f_+ + \beta f_-\|_{L_p(\mathcal{T})}.$$

Let $\mathcal{S}_1(\mathcal{T}) := \{g(\mathbf{x}) = \mathbf{a}\mathbf{x}^t + c : \mathbf{a} \in \mathbb{R}^d, c \in \mathbb{R}, \mathbf{x} \in \mathcal{T}\}$. For $f \in L_p(\mathcal{T})$, set

$$E(f; \mathcal{S}_1(\mathcal{T}))_{L_{p;\alpha,\beta}(\mathcal{T})} := \inf\{\|f - u\|_{L_{p;\alpha,\beta}(\mathcal{T})} : u \in \mathcal{S}_1(\mathcal{T})\}.$$

Note that $E(f; \mathcal{S}_1(\mathcal{T}))_{L_{p;1,1}(\mathcal{T})}$ is the usual best approximation of f by linear functions on $\mathcal{T}$ in the L_p-metric. Let $Q(\mathbf{x}) = \sum_{j=1}^{d} x_j^2$. We solve the following extremal problem:

$$E(Q; \mathcal{S}_1(\mathcal{T}))_{L_{p;\alpha,\beta}(\mathcal{T})} \to \inf, \quad |\mathcal{T}| = 1, \tag{1}$$

where $|\mathcal{T}|$ stands for the volume of simplex $\mathcal{T}$.

To the best of our knowledge, problem (1) was solved only in the case of one-sided approximation (interpolation) and in the case $\alpha = \beta = 1$, $p = 2$ and $d = 2$. Problem (1) is important for many questions in Conves Geometry, approximation of convex bodies by polytopes and approximation of surfaces by splines. Let $\mathcal{T}_0$ be the regular simplex of unit volume in $\mathbb{R}^d$. We obtained the following result.

Theorem. *Let $\alpha, \beta > 0$, $d \in \mathbb{N}$ and $1 \le p \le \infty$. Then*

$$E(Q; \mathcal{S}_1(\mathcal{T}_0))_{L_{p;\alpha,\beta}(\mathcal{T}_0)} = \inf_{\mathcal{T} : |\mathcal{T}|=1} E(Q; \mathcal{S}_1(\mathcal{T}))_{L_{p;\alpha,\beta}(\mathcal{T})}.$$

EXACT ASYMPTOTICS OF THE OPTIMAL L_p-ERROR OF ASYMMETRIC LINEAR SPLINE APPROXIMATION

V.F. Babenko[1], **Yu.V. Babenko**[2], **N.V. Parfinovych**[1], **D.S. Skorokhodov**[1]

[1] Dnepropetrovsk National University, UKRAINE,
[2] Sam Houston State University, Huntsville, TX, USA
babenko.vladislav@gmail.com, YVB001@shsu.edu, nparfinovich@yandex.ru, dmitriy.skorokhodov@gmail.com

Let $D = [0, 1]^2$. By L_p, $1 \le p \le \infty$, denote the space of functions $f : D \to \mathbb{R}$ endowed with the usual norm $\|\cdot\|_p$. Let α and β be positive continuous on D functions, and let $f \in L_p$. Then, asymmetric L_p-norm is defined as follows

$$\|f\|_{L_{p;\alpha,\beta}(D)} := \|\alpha f_+ + \beta f_-\|_p,$$

where $g_\pm(\cdot) = \max\{g(\cdot); 0\}$.

A collection $\triangle_N = \triangle_N(D) = \{T_i\}_{i=1}^N$ of N triangles, $N \in \mathbb{N}$, in the plane is called a *triangulation* of a set D provided that: any pair of triangles from $\triangle_N$ intersects at most at a common vertex or along a common edge, and $D = \bigcup_{i=1}^{N} T_i$.

Given a triangulation $\triangle_N$, let $\mathcal{S}(\triangle_N)$ be the space of continuous on D functions, which are linear on every triangle $T_i \in \triangle_N$. Now set

$$R_N(f, L_{p;\alpha,\beta}) := \inf_{\triangle_N} \inf_{s\in\mathcal{S}(\triangle_N)} \|f - s\|_{L_{p;\alpha,\beta}(D)},$$

where the first inf is taken over all triangulations with N triangles.

Let T_0 be the equilateral triangle. For $f \in C^2(D)$, set $H(f;\cdot) := \frac{\partial^2 f}{\partial x^2}(\cdot)\frac{\partial^2 f}{\partial y^2}(\cdot) - \left(\frac{\partial^2 f}{\partial xy}(\cdot)\right)^2$. Let

$$C_{p;\alpha,\beta}(x,y) := \inf_{a,b,c\in\mathbb{R}} \|u^2 + v^2 - au - bv - c\|_{L_{p;\alpha(x,y),\beta(x,y)}(T_0)}.$$

Theorem. *Let $f \in C^2(D)$; $H(f; x, y) \geq C^+ > 0$ for all $(x, y) \in D$. Let positive continuous on D functions α and β also be given. Then for all $1 \leq p < \infty$,*

$$\lim_{N\to\infty} N \cdot R_N(f, L_{p;\alpha,\beta}) = \left(\int_D H^{\frac{p}{2(p+1)}}(f; x, y) C_{p;\alpha,\beta}^{\frac{p}{p+1}}(x, y)\, dxdy\right)^{\frac{p+1}{p}}.$$

ESTIMATES OF MODULUS OF CONTINUITY FOR FUNCTION FROM BESOV-TYPE SPACE

Eugeny Berezhnoi
Yaroslavl State University, Yaroslavl, Russia
ber@uniyar.ac.ru

Let X be a symmetric function space on $I = [0; 1]^n$, $\psi(X, t) = \|\chi([0, t^{1/n}]^n)|X\|$ - is a fundamental function of X and

$$\omega(f, h; X) = \sup_{0\leq|\delta|\leq h} \|f(. + \delta) - f(.)|X\|$$

is a modulus of continuity of function f. Let $\Lambda^{\alpha}_{X,p}$ is a Besov-type space with norm

$$\|f|\Lambda^{\alpha}_{X,p}\| = (\sum_{i=1}^{\infty}(2^{\alpha i} \cdot \omega(f, 2^{-i}; X))^p)^{1/p}, \quad (\alpha \in (0,1), p \in [1,\infty)).$$

Theorem. *Let $\epsilon > 0$, X be a symmetric space, $f \in \Lambda^{\alpha}_{X,p}$ and a sequence positive numbers$\{\epsilon_i \downarrow 0\}$ such that*

$$\sum_{i=1}^{\infty}\epsilon_i < \epsilon, \quad \sum_{i=1}^{\infty}\left(\frac{2^{-\alpha\cdot i/p}}{\psi(X;\epsilon_i)}\right)^q < \infty, \quad (\frac{1}{p}+\frac{1}{q}=1).$$

Let is

$$\Omega(h) = \inf_k\{h\left(\sum_{i=1}^{k}\left(2^i\cdot\frac{2^{-\alpha\cdot i/p}}{\psi(X;\epsilon_i)}\right)^q + \sum_{i=k+1}^{\infty}\left(\frac{2^{-\alpha\cdot i/p}}{\psi(X;\epsilon_i)}\right)^q\right)^{1/q}\}.$$

There exists the set $W(\epsilon) \subset I$ and $c > 0$ such that

$$\mu(W(\epsilon)) < \epsilon,$$

and if $x, y \in I\backslash W(\epsilon)$, then

$$|f(x) - f(y)| \leq c\cdot\Omega(|x-y|)\cdot\|f|\Lambda^{\alpha}_{X,p}\|,$$

so c not depend from ϵ and f.

Author had proved analog of this theorem for function with Holder condition before.

The author was supported by grant RFFI, N 08-01-00669.

FRACTAL HARMONIC WAVELETS

Carlo Cattani
University of Salerno (SA), Italy
ccattani@unisa.it

Fractal Harmonic Wavelets (FHW) are special class of fractal-like bases, defined on the so-called Harmonic Wavelets. Harmonic wavelets are complex values wavelets, of Littlewood-Paley type, with sharp compact support in Frequency domain. The real part of these functions give rise to the Shannon Wavelets multiresolution analysis (based on the well-known sinc-function) which can be considered as based on the infinite degree interpolating functions. It is by using a finite degree interpolation (Fractal Interpolating Functions, FIF), that Hardin and Massopust (and later Geronimo et Al.) started in 90ies the theory of fractal wavelets. The main fractal properties (self-similarity, fractal dimension, nowhere differentiability) of FHW are shown. Moreover, the reconstruction of fractals, by using FHW, is explicitly given in some examples. In particular, the generalized Riemann-Weierstrass function is reconstructed in the FHW theory and recognized as a particular case thereof.

References

1. C.Cattani, Harmonic Wavelet Approximation of Random, Fractal and High Frequency Signals, to appear on Telecommunication Systems (2009).

2. C.Cattani, Wavelet Analysis of Fractals, to appear on Bulletin of the Chelyabinsk State University (2009).

3. C. Cattani, Shannon Wavelets Theory, Mathematical Problems in Engineering, vol. 2008, Article ID 164808, 24 pages, (2008), doi:10.1155/2008/164808

4. Cattani and J.J. Rushchitsky, Wavelet and Wave Analysis as applied to Materials with Micro or Nanostructure, Series on Advances in Mathematics for Applied Sciences, World Scientific, Singapore, 74 (2007).

5. D.P.Hardin, B. Kessler and P. Massopust, Multiresolution Analysis based on Fractal Functions, J. Approx. Theory, 71, (1992) 104-120.

6. D.P.Hardin, and P. Massopust, The capacity for a class of Fractal Functions, Comm. Math. Phys., 105, (1986) 455-460.

7. D.E. Newland, Harmonic wavelet analysis, Proc.R.Soc.Lond. A, 443, (1993) 203-222.

APPLICATION OF WAVELETS FOR SOLVING OF BOUNDARY PROBLEMS

N.I. Chernykh, Yu.N. Subbotin
IMM UB RAN, Ekaterinburg, Russia
Chernykh@imm.uran.ru, yunsub@imm.uran.ru

In Il'in's papers [1] for solutions of a boundary-value Dirichlet problem in doubly-connected domains for elliptic partial differential equations, the asymptotic expansion with respect to a small parameter was given, when the small parameter is the cross dimension of a slit contracting to a segment. The expansion is series in fractional powers and powers logarithmic functions of the small parameter. In this report on the results of [2] a basis of harmonic wavelets is constructed in an elliptic ring and its approximation properties are investigated (see also [3]). The obtained results are used to analyze the behavior of the solution $U(\zeta, \varphi_t, \varphi_R)$ of a boundary-value Dirichlet problem $\{\Delta U = f$ in $\mathbf{E}(t,R),\ U|_{\Gamma_R} = U_R(\zeta),\ U|_{\Gamma_t} = U_t(\zeta)\}$ in a domains $\mathbf{E}(t,R)$ between two confocal ellipses $\Gamma_t,\ \Gamma_R$, where Γ_τ is $\frac{\xi^2}{((\tau+\tau^{-1})/2)^2} + \frac{\eta^2}{((\tau-\tau^{-1})/2)^2} = 1$, $1 \le t < R$, under the contraction of the contour Γ_t to the slit Γ_1. The received expansion of the solution $U(\zeta, \varphi_t, \varphi_R)$ on harmonic wavelets in $\mathbf{E}(t,R)$ converges uniformly in $\overline{\mathbf{E}(t,R)}$ $(1 \le t < R)$, and none of the series terms depends critically on the small parameter $t-1 \to 0$ (do not converges to 0 or ∞ as $1 \le t \le \tau \to 1$ for the general functions $\varphi_R(\zeta)$; hence, none of these elements is infinitely small with respect to another as $t \to 1$). The expansion of $U(\zeta, \varphi_t, \varphi_R)$ does not involve, as in [1], terms with negative degrees of the functions $\sqrt{1-\zeta^2}$ and $\ln|\sqrt{1-\zeta^2}|$. It can be differentiated along the lines Γ_τ for all $\tau \in [1, R)$ termwise except for the endpoints of the cut for $\tau = t = 1$. The corresponding expansion of derivative converges in domain $\overline{\mathbf{E}(t,R)}$ for $t > 1$. Such derivative $\partial_t U(\zeta, \varphi_t, \varphi_R)\Big|_{\zeta\in\Gamma_\tau}$ grows unboundedly, the order of growth being $O(1/|\sqrt{\zeta^2-1}|)$, only for ζ tending to the ends of the cut.

This work was supported by the RFBR (08-01-00320, and 09-01-00014), by the Program of OMS "Contemporary problem of mathematics 2009.

References

1. A.M.Il'in. Mat. Sb. 1976. **99(141)** (4), 514.

2. Yu.N.Subbotin, N.I.Chernykh. Mat. Model. 2002. **14** (5), 17.

3. Yu.N.Subbotin, N.I.Chernykh. Proc. of the Steklov Inst. of Math. 2009. Suppl. 2. P. S103–S117. (to be appear).

APPROXIMATIVE PROPERTIES OF WAVELETS WHICH ARE INTERPOLATING AND ORTHOGONAL

N.I. Chernykh, Yu.N. Subbotin

IMM UB RAN, Ekaterinburg, Russia

Chernykh@imm.uran.ru, yunsub@imm.uran.ru

In the report new systems of periodic wavelets and wavelets on the whole axis are constructed (based upon Meyer wavelets); these systems are orthogonal and interpolating simultaneously. Two methods are proposed of constructing the required functions. Let $\widehat{\varphi}(\omega) = \widehat{\varphi}_\varepsilon(\omega)$ be the arbitrary Meyer-type functions supported on $[-(1+\varepsilon)/2, (1+\varepsilon)/2]$, $0 < \varepsilon \le 1/3$ (see [1]), and $\widehat{\varphi}_1(\omega) = \widehat{\varphi}(\omega) + (1-\widehat{\varphi}(\omega)-\widehat{\varphi}(\omega-1))/2 + i\,(\operatorname{sign}\omega)\sqrt{\widehat{\varphi}(\omega)\widehat{\varphi}(\omega-1)/2}$, $\widehat{\varphi}_2(\omega) = |\widehat{\varphi}(\omega)|^2 + i\,(\operatorname{sign}\omega)\widehat{\varphi}(\omega)\widehat{\varphi}(\omega-1)$. The corresponding function $\varphi_s(x)$ $(s = 1,2)$ generate the orthonormal bases $\{\varphi_{s,j,k}(x) = 2^{j/2}\varphi_s(2^j x - k)\}_{k\in\mathbf{Z}}$ of the subspaces $\{V_{s,j}\}_{j\in\mathbf{Z}}$, which constitute the multiresolution analysis of the space $L^2(\mathbf{R})$. This systems are simultaneously interpolating on the grid $\{x_{j,r} = r/2^j :\ r \in \mathbf{Z}\}$ in the sense that $2^{-j/2}\varphi_{s,j,k}(x_{j,r}) = \delta_{r,k}$ $(r,k \in \mathbf{Z})$. The corresponding wavelets $\psi_s(x)$ can be defined as usual by $\widehat{\psi}_s(\omega) = e^{i\pi\omega} m_s((\omega+1)/2)\widehat{\varphi}_s(\omega/2)$ $(s = 1,2)$, the system $\{2^{-j/2}\psi_{s,j,k}(x)\}_{j,k\in\mathbf{Z}}$ is interpolating on the grid $\left\{\frac{2k-1}{2^{j+1}} :\ j,k \in \mathbf{Z}\right\}$.

Denote by $\Phi_{s,j,k}(x)$ and $\Psi_{s,j,k}(x)$ $(j,k \in \mathbf{Z}_+,\ 1 \le k \le 2^j)$ the trigonometric polynomial of order $[2^{j-1}(1+\varepsilon)]$, $\Phi_{s,j,k}(x) = \sum\limits_{\nu\in\mathbf{Z}} \varphi_{s,j,k}(x+\nu)$, $\Psi_{s,j,k}(x) = \sum\limits_{\nu\in\mathbf{Z}} \psi_{s,j,k}(x+\nu)$, The system $\{2^{-j/2}\Psi_{s,j,k}(x) :\ j,k \in \mathbf{N},\ 1 \le k \le 2^j\}$ is also interpolating on the grid $\left\{\frac{2l-1}{2^{j+1}} :\ j \in \mathbf{Z}_+, l \in \mathbf{Z}\right\}$.

Estimates of the errors of approximation in the Chebyshev norm different classes of smooth functions by these wavelets will be formulated in the report.

On the Dubuk and Evangelista orthonormal-interpolating wavelets see in [2].

This work was supported by the RFBR (08-01-00213, and 09-01-00014), by the Program of OMS "Contemporary problem of mathematics 2009.

References

1. D.Offin and K.Oskolkov. Constr. Approx. 1993. **9**, 319.
2. I.Dobeshi. *Ten Lectures about Wavelets* (Regulyarn. i Khaotich. Dinamika, Izhevsk, 2001) [in Russian].

HYPERSPECTRAL IMAGING AND DIMENSIONALITY REDUCTION

Charles Chui
University of Missouri - St. Louis and Stanford University
ckchuistanford.edu

Since human vision is restricted to electromagnetic radiation in the frequency band of 400 – 790 terahertz (1 THz = 10^12 cycles per second), wavelengths of visible light to the human eye are limited to the range of 400 – 700 nm (1 meter = 10^9 nanometers). On the other hand, with 12 color channels and complex structure, the eyes of a mantis shrimp have the capability of viewing electromagnetic radiation ranging from ultraviolet (UV: 10 nm – 400 nm) to some range of the infrared spectrum (Near IR: 700 nm – 1,000 nm). In other words, a mantis shrimp has the so-called “hyperspectral” vision. With the recent rapid advances of satellite, senor, and computing technologies, it is now feasible to capture, render, and (off-line) analyze complex hyperspectral image (HSI) data of sufficiently high resolution. We will discuss the “what and why” of hyperspectral imaging, and briefly mention a wide spectrum of its practical applications: from agricultural monitoring to geospatial mapping, and from security surveillance to cancerous tissue detection in medical imaging. However, the mathematics of HSI is most challenging and requires innovative ideas, since the data-set of a typical HSI cube is quite large. For instance, the data kernel of a one megapixel-resolution HSI with 200 spectral bands is a 10^6x200 matrix. Hence, a very important problem is to be able to reduce the kernel size while preserving the important data information, such as manifold geometry and topology. This problem is called "dimensionality reduction" of HSI data. Unfortunately, linear methods such as principal component analysis (PCA) and multi-dimensional scaling (MDS) are not effective for the study of dimensionality reduction of HSI data. On the other hand, to apply such powerful mathematical tools as diffusion maps and diffusion wavelets, the first step is to symmetrize the HSI data matrix, resulting in square matrices of very high dimensions, governed by the image resolution instead of the number of spectral bands. I will briefly discuss the general architecture of the current non-linear methods for dimensionality reduction to achieve symmetry, the various difficulties that arise, and the defects of current solutions to overcome such difficulties, particularly in terms of neighborhood selection and data-set tiling. I will then discuss the approach that my collaborator, Jianzhong Wang, and I recently introduced and end my talk by demonstrating our methods with some experimental results.

WAVELETS ON A MANIFOLD

Yuri K. Dem'yanovich
Faculty of Mathematics and Mechanics,
St.-Petersburg State University, Russia
Yuri.Demjanovich@JD16531.spb.edu

Numerical flows associated with a smooth manifold can be processed using local functions (see [1]). However, wavelet decompositions have to be invoked to develop efficient algorithms.

The goal of this paper is to describe the scheme for constructing wavelets based on approximation relations. Specifically, conditions for embedding spaces of local functions are presented, a wavelet decomposition is constructed, the corresponding decomposition/reconstruction formulas are derived, the number of operations in them is assessed, and the order of smallness of the wavelet component is estimated in terms of the approximation order. The classical approaches to the construction of wavelets are based on the Fourier transform or the lifting scheme (see [2-6]). In this paper, we start from approximation relations (see [7-8]). As a result, the wavelet decompositions constructed have an order of approximation that is asymptotically optimal with respect to the N-width of standard compact sets. Moreover, the coordinate wavelets are smooth, and they have a compact support of minimal (index) length (for a given approximation order). The coefficient in the linear dependence of the computational complexity on the amount of input data is easily estimated in terms of the approximation order.

This work was supported by the Russian Foundation for Basic Research, project nos. 07-01-00451 and 07- 01-00269.

References

1. Yu.K.Dem'yanovich, Local Approximations on Manifolds and Minimal Splines (St.-Petersburg Univ., St.-Petersburg, 1994) [in Russian].

2. S.Mallat, Wavelet Tour of Signal Processing (Academic, New York, 1999; Mir, Moscow, 2005).

3. I.Ya.Novikov, V.Yu.Protasov and M.A.Skopina, Theory of Wavelets (Fizmatlit, Moscow, 2005) [in Russian].

4. I.Daubechies, I.Guskov, and W.Sweldens, Constructive Approximation, No. 15, 381-426 (2001).

5. W.Sweldens, J.Appl. Comput. Harmonic Anal., No.3(2),186-200 (1996).

6. E.E.Tyrtyshnikov, Numerical Analysis Methods (Moscow, 2007) [in Russian].

7. Yu.K.Dem'yanovich, Dokl. Math. 65, 47.50 (2002) [Dokl. Akad. Nauk 382, 314-319 (2002)].

8. Yu.K.Dem'yanovich, Dokl.Math.79,1-4 (2009) [Dokl. Akad. Nauk 401, 438-442 (2005)].

О ПРИБЛИЖЕНИИ ПЕРИОДИЧЕСКИХ ФУНКЦИЙ СИНГУЛЯРНЫМИ ИНТЕГРАЛАМИ С ПОЛОЖИТЕЛЬНЫМИ ЯДРАМИ.

Н. Ю. Додонов

Санкт-Петербургский государственный университет

dodonov@math.spbu.ru

Для пространств L_p и C, состоящих из 2π-периодических функций двух переменных, рассматривается следующий вопрос: как связано поведение величин

$$\left\| \int\limits_a^b \int\limits_c^d \Delta^r_{u,v}(f;\cdot)\varphi_n(u)\psi_m(v)dudv \right\|_p$$

при $n, m \to \infty$, где $a \geq 0, c \geq 0$,
$\Delta^r_{u,v}(f;x,y) = \sum\limits_{i,j=1}^{r}(-1)^{i+j}C^i_rC^j_rf(x+iu,y+jv) - f(x,y)$,
φ_n, ψ_m - положительные ядра, со структурными свойствами функции f, характеризуемыми посредством её модулей непрерывности.

Приведём пример установленных результатов. Через

$$\omega_1(f;u,v)_p = \sup_{|t|\leq u, |h|\leq v} \|f(x+t,y+h) - f(x,y)\|_p$$

обозначаем модуль непрерывности первого порядка функции f в пространстве L_p,

$$\Phi_n(t) = \frac{1}{2\pi n}\left(\frac{\sin\frac{nt}{2}}{\sin\frac{t}{2}}\right)^2$$

— ядро Фейера.

Теорема. Пусть $1 \leq p \leq \infty$, $f \in L_p$ при $1 \leq p < \infty$, $f \in C$ при $p = \infty$,

$$\sigma^*_{n,m}(f;x,y) = 4\int\limits_0^\pi \int\limits_0^\pi f(x+u,y+v)\Phi_n(u)\Phi_m(v)dudv.$$

Тогда выполнение совокупности соотношений

$$\varliminf_{n\to\infty}\frac{\pi n}{2\ln n}\varliminf_{m\to\infty}\|f - \sigma^*_{n,m}(f)\|_p \leq K,$$

$$\varlimsup_{m\to\infty}\frac{\pi m}{2\ln m}\varlimsup_{n\to\infty}\|f-\sigma^*_{n,m}(f)\|_p\le L$$

равносильно выполнению неравенства

$$\omega_1(f;u,v)_p\le Ku+Lv \qquad (u,v\in\mathbb{R}_+).$$

Литература

1. Додонов Н. Ю., Жук В. В. О приближении периодических функций сингулярными интегралами с положительными ядрами // Зап. научн. семинаров ПОМИ. 2006. Т. 337. С. 51-72.

2. Додонов Н. Ю., Жук В. В. Об аппроксимативных характеристиках функций многих переменных, входящих в классы насыщения модулей непрерывности различных порядков // Проблемы математического анализа. 2006. Вып. 33. С. 79-90.

p-ADIC MULTIRESOLUTION ANALYSES, WAVELET BASES AND FRAMES

Sergei Evdokimov

St.Petersburg Department of Steklov Mathematical Institute
Russian Academy of Sciences, St.Petersburg, Russia
evdokim@pdmi.ras.ru

We review the contents of papers [1–5]. This in particular includes the definition of a p-adic multiresolution analysis (MRA) and the statement that any orthogonal scaling test function generating such an MRA is 1-periodical and, moreover, in contrast to the real case generates in fact the Haar MRA. Furthermore, within the framework of the MRA theory we describe a general technique of constructing wavelet systems that form frames and Riesz bases in $L^2(\mathbf{Q}_p)$. A realization of the technique is presented. Namely, we construct an infinite family of mutually distinct MRA's each of which is generated by a non-1-periodical (and consequently non-orthogonal) scaling test function, and show that this leads to an infinite family of mutually distinct non-orthogonal wavelet Riesz bases.

References

1. S.Albeverio, S.Evdokimov and M.Skopina, *p-Adic multiresolution analysis and wavelet frames*, 2008, 1-15, submitted to Journal of Fourier Analysis and Applications, URL: http://arxiv.org/abs/0810.1147.

2. Evdokimov S.A. and Skopina M.A., 2*-Adic wavelet bases*, Proceedings of Institute of Mathematics and Mechanics, Ural Branch of Russian Academy of Sciences, 15 (2009), no. 1, 135-146 (in Russian).

3. S.Albeverio, S.Evdokimov and M.Skopina, *p-Adic non-orthogonal wavelet bases*, accepted for publication in Proceedings of International Conference on p-Adic Mathematical Physics, October 1-6, 2007, Moscow, Russian Federation.

4. V. M. Shelkovich, M. Skopina, *p-Adic Haar multiresolution analysis and pseudo-differential operators*, accepted for publication in Journal of Fourier Analysis and Applications (2009).

5. A.Yu.Khrennikov, V.M.Shelkovich and M.Skopina, *p-Adic refinable functions and MRA-based wavelets*, accepted for publication in Journal of Approximation Theory (2009).

WAVELETS AND FRAMES ON THE CANTOR DYADIC GROUP

Yu.A. Farkov

Russian State Geological Prospecting University, Russia

farkov@list.ru

The Cantor dyadic group $\mathcal{C}$ is defined to be the locally compact abelian group formed by taking the weak cartesian product of a countable many copies of the discrete cyclic group $\mathbf{Z}_2$ with the product of topologies. It is well-known that Walsh functions are characters for $\mathcal{C}$ (see, e.g., [1]). Orthogonal compactly supported wavelets on $\mathcal{C}$ were introduced in 1996 by W.C. Lang. Some further results and the detailed bibliographi on this subject are given in [2], [3]. In this talk, we present several examples of frames for $\mathcal{C}$.

References

1. F. Schipp, W.R. Wade, and P. Simon, Walsh series: An introduction to dyadic harmonic analysis, Adam Hilger, Bristol and New York, 1990.
2. Yu.A. Farkov, Multiresolution analysis and wavelets on Vilenkin groups, Facta Univers. (NIŠ). Ser.: Elec. Energ. **21** : 3 (2008), 309-325.
3. Yu.A. Farkov, Biorthogonal wavelets on Vilenkin groups, Proceedings of the Steklov Institute of Mathematics **265** (in press).

LAYER POTENTIALS AND WAVELETS ON REGULAR SURFACES

Willi Freeden
TU Kaiserslautern, Geomathematics Group, Germany
freeden@mathematik.uni-kl.del

The talk is concerned with wavelets on regular surfaces such as sphere, ellipsoid, geoid, Earths surface, etc. By means of the limit and jump relations of classical potential theory the wavelet approach is explained in detail. The properties of a multiresolution analysis are verified by explicitly available potential kernel representations, and a tree algorithm for fast computation is developed based on numerical integration. As applications of the potential theoretic wavelet approach some numerical examples are presented, including the zoom-in property of high frequency perturbations.

Finally fast multiscale representations of the solution of boundary value problems are discussed for several types of differential equations, viz. the Helmholtz equation, the Cauchy-Navier-equations of elasticity, the Maxwell equations of electromagnetic theory, and the Stokes equations of fluid dynamics.

References

1. W. Freeden (1980). On the Approximation of External Gravitational Potential With Closed Systems of (Trial) Functions. Bull. Geod., 54, 1-20.

2. Freeden, W., Mayer, C. (2003). Wavelets Generated by Layer Potentials. Appl. Comp. Harm. Anal. (ACHA), 14, 195-237.

3. Freeden, W., Mayer, C., and Schreiner, M. (2003). Tree Algorithms in Wavelet Approximation by Helmholtz Potential Operators. Numer. Funct. Analysis and Optimization, 24, 747-782.

4. Abeyratne, M.K., Freeden, W., Mayer, C (2003). Multiscale Deformation Analysis by Cauchy-Navier Wavelets. J. Appl. Math., 12, 605-645.

5. Freeden, W., Mayer, C. (2006). Multiscale Solution for the Molodensky Problem on Regular Telluroidal Surfaces. Acta Geod. Geoph. Hung.,41, 55-86.

6. Freeden, W., Mayer, C. (2007). Modeling Tangential Vector Fields on Regular Surfaces by Means of Mie Potentials. Int. J. Wavelets, Multiresolution and Information Processing, 5, 417 – 449.

NEW VARIANT OF WAVELETS DECOMPOSITIONS FOR SPLINE SPACES

Mohamed Waleed C. O. Gabr
St. Petersburg, Russia
mwaleed73@yahoo.com

In previous researches (see [1-3]) for construction wavelet decomposition biorthogonal system of functionals, defined by the derivatives of the generating function was used. However, in cases where the values of the derivatives of the generating functions are not known, should be limited to only the values of the function itself. In this paper, biorthogonal system is defined by means of differences, that allows to construct wavelets decomposition, using only the value of the mentioned function. As a result the formulas of decompositions and reconstruction are deduced; without using the derivatives of the functions which generate numerical flows.

References

1. Yu. K. Dem'yanovich, "Wavelet decompositions on a nonuniform grid"[in Russian], Trudy S.-Peterburg. Mat. O-va, 13 (2007), 27-51.
2. Yu. K. Dem'yanovich, "Minimal Splines and Wavelets "Vestnik St. Petersburg University. Mathematics, 2008, Vol. 41, No. 2, pp. 88-101.
3. Yu. K. Dem'yanovich, "Wavelets and Minimal Splines"[in Russian], St. Petersburg Univ., 2003.
4. de Boor, C., A Practical Guide to Splines, Springer Verlag, New York, 1978.

NONSEPARABLE WAVELETS OF MEYER TYPE IN BESOV AND LIZORKIN-TRIEBEL SPACES [1]

S. A. Garkovskaya

Voronezh State University, Russia

GarkovskayaSA@mail.ru

Nonseparable wavelets of Meyer type in $\mathbb{R}^2$ associated to the dilation matrix $M := \begin{pmatrix} 0 & -1 \\ 2 & 1 \end{pmatrix}$ were constructed in [1]. It is proved that Fourier transforms of nonseparable wavelets of Meyer type, associated to the dilation matrix M, can be used as decomposition of unity in definition of Besov and Lizorkin-Triebel spaces. The result is the first step in the proof of unconditional basisness of above mentioned wavelets in scales under consideration. The basisness of separable wavelets of Meyer-David type is investigated in [2, c. 498] . The proof of the basisness of nonseparable wavelets in Besov spaces can be found in [3].

Theorem: *Let φ be the scaling function for the multiresolution analysis associated to the dilation matrix $M = \begin{pmatrix} 0 & -1 \\ 2 & 1 \end{pmatrix}$, $\psi \in L_2(\mathbb{R}^2)$*

[1]This work is supported by RFBR (grant 08-01-00226-a).

— is a Meyer type wavelet associated to M. Let $\nu = \{\nu_j(\xi)\}_{j=0}^{\infty}$, where $\nu_0(\xi) = \widehat{\varphi}(\xi)$, $\nu_j(\xi) = \widehat{\psi}((M^*)^{-(j-1)}\xi)$ ÏaË $j = 1, 2, 3, \ldots$.

(i). Let $-\infty < s < \infty$, $1 < q \leqslant \infty$, $1 < p \leqslant \infty$. Then

$$\|f|B^s_{p,q}(\mathbb{R}^2)\|^\nu = \|2^{sj}F^{-1}\nu_j Ff|l_q(L_p(\mathbb{R}^2))\|$$

are equivalent norms in $B^s_{p,q}(\mathbb{R}^2)$.

(ii). Let $-\infty < s < \infty$, $1 < q \leqslant \infty$, $1 < p < \infty$. ’®J¤

$$\|f|F^s_{p,q}(\mathbb{R}^2)\|^\nu = \|2^{sj}F^{-1}\nu_j Ff|L_p(l_q, \mathbb{R}^2)\|$$

are equivalent norms in $F^s_{p,q}(\mathbb{R}^2)$.

References

1. M. Bownik, D. Speegle Meyer Type Wavelet Bases in $\mathbb{R}^2$, Journal of Approximation Theory 116 (2002), pp.49-75.

2. I.Ya. Novikov, V. Yu. Protasov, M.A. Skopina Theory of Wavelets. - M.: Physmatlit, 2005. - 616p.

3. M. Lindemann Approximation Properties of Non-Separable Wavelet Bases with Isotropic Scaling Matrices and their Relations to Besov Spaces, University Bremen, 2005.

PAIRS OF DUAL WAVELET FRAMES IN VARIOUS FUNCTION AND DISTRIBUTION SPACES

Bin Han
University of Alberta
bhan@math.ualberta.ca

In this talk, we shall discuss wavelets under the framework of pairs of dual wavelet frames in various function and distribution spaces. We fully characterize dual wavelet frames in distribution spaces, which allows us to naturally connect wavelets in function spaces to the fast wavelet transform in the discrete setting. This also enables us to understand better many basic properties of traditional wavelets. Next, we shall discuss dual wavelet frames in various function spaces, in particular, Sobolev spaces. Wavelet frames and Riesz wavelets in Sobolev spaces are of interest in numerical algorithms and image processing. The traditional approach is to obtain wavelets in $L_2(R^d)$, and then to extend such wavelets to certain Sobolev spaces. This approach excludes many interesting wavelets in Sobolev spaces. In this talk, we shall present a natural framework to study dual wavelet frames and Riesz wavelets in a pair of Sobolev spaces $(H^s(R^d), H^{-s}(R^d))$ for any real number s. The fundamental norm equivalence property of wavelet expansions for functions in Sobolev spaces will become a natural byproduct of our approach. Examples will be provided to illustrate our results. For example, we show that $\{2^{j(1/2-s)}B_m(2^j \cdot -k) \ : \ j \in N_0, k \in Z\}$ is a wavelet frame in $H^s(R)$ for any $0 < s < m - 1/2$, where B_m is the B-spline of order m. This is also true for a large class of refinable functions (no stability is required) including almost all box splines in any dimension. If time permits, recent developments on symmetric orthonormal complex wavelets, which are particular families of dual wavelet frames, will also be mentioned in this talk.

COMPARISON CONTINUES WAVELET TRANSFORM AND DISCRETE WAVELET TRANSFORM IN SIGNAL FEATURE EXTRACTION

Ali Ghaffari, Ensieh Sadat Hosseini Rooteh
Dept. of Mechanical Engineering, Khaje Nasir Toosi University, Tehran, Iran
ghaffari@kntu.ac.ir, ensiehhoseini@gmail.com

Wavelet transform is one of the high power signal processing and feature extraction instruments. Wavelet transform is a common method for feature extraction in pattern recognition field. In this study some heart diseases are diagnosed by electrocardiogram signal analysis. Heart disease detection has two steps. In the first step heart signal features are extracted by continues wavelet transform and discrete wavelet transform. Neural networks are used in second step for feature vector classifying. Finally by comparing the result of heart disease detection with two methods of feature extractions, the ability of continues wavelet transform and discrete wavelet transform methods in signal processing and feature extractions are compared.

ALGEBRAS WITH WAVELETS, REPRODUCING KERNELS, DELTA FUNCTIONS.

S. Gritsutenko
Omsk State Transport University, Russia
st256@mail.ru

Firstly Hilbert Space is defined. Then the convolution is defined there. Then delta vector is defined there. Some features of delta vector are proved (for example, that such delta vector is the Mother Wavelet). Other words, method for building of wavelets in arbitrary Hilbert space is described in this address.

REVERSE TIME MIGRATION USING WAVELETS FOR DATA REDUCTION

Maxim Ilyasov
Germany
ilyasov@itwm.fhg.de

Seismic migration methods plays a prominant role in the geophysical science, for example in the oil or gas exploration. Reverse time migration (RTM) based on the full-way wave equation is such the method, that able to yield fine interior structural details, despite certain deficiency of initial values.

However, the main disadvantage of RTM, that makes this method in most of cases unusable, is the high computational requirements and I/O, for saving intermediate wave-field propagations, that are needed to calculate result image.

The main goal of this work is to modify the classical RTM. To reduce the high volumes of computational data, we use checkpoint technique, that saves intermediate data at the rough grid. To define the wave-field at the fine grid we apply the wavelet-approach [Freeden & Schreiner, 2009], that can easily reconstruct this wave-field with the given accuracy locally. The advantage of this approach is reduction of unwanted internal reflections and noise. It makes also possible to apply the advanced imaging condition [Liu et al. 2007].

All this techniques produce images of reflectors with less noise than classical RTM and significantly reduce the computational time.

References

1. E. Baysal, D.D. Kosloff, J.W.C. Sherwood, Reverse time migration: Geophysics, Vol. 48, No. 11, 1983, p. 1514-1524
2. E. Baysal, D.D. Kosloff, J.W.C. Sherwood, A two-way nonreflecting wave equation: Geophysics, Vol. 49, No. 2, 1984, p. 132-141
3. W. Freeden, M. Schreiner, Spherical Functions of Mathematical Geosciences: A Scalar, Vectorial, and Tensorial Setup (Advances in Geophysical and Environmental Mechanics and Math), 2009
4. F. Liu, G. Zhang, S.A. Morton, J.P.Leveille, Reverse-time Migration Using One-way Wavefield Imaging Condition: SEG Annual Meeting, SanAntonio, 2007
5. O. Yilmaz,Seismic Data Processing: SEG, Tulsa, 2001

ON THE CONNECTION BETWEEN THE CONTINUITY MODULE WITH PIECEWISE-POLYNOMIAL APPROXIMATIONS

Irina Irodova

Yaroslavl Demidov State University, Russia

IrinaIrodova@gmail.com

Let Q_0 be a unit cube in d-dimensional space. We will call the partition F_n of cube Q_0 "almost" dyadic, if the partition F_n consists of parallelepipeds with edges of length $\approx 2^{-n}$ and the partition F_{n+1} is a refinement of the partition F_n.

Let P_k be a space of polynomials of degree $k-1$ for each variable. Let us denote by $P_k(\Pi)$ a space of piecewise-polynomial functions, subordinated to partition Π of cube Q_0. Then $e_k(f,\Pi)_p$ - a distance from f befor $P_k(\Pi)$ in $L_p(Q_0)$.

The next result allows us to compare the degree of approximation by piecewise-polynomial functions in different norms.

Theorem 1. *Let $f \in L_p(Q_0), 0 < p < q < \infty$, then*

$$e_k(f,F_n)_q \leq c \cdot \left(\sum_{i=n}^{\infty} \left(2^{id\left(\frac{1}{p}-\frac{1}{q}\right)} e_k(f,F_i)_p \right)^q \right)^{\frac{1}{q}},$$

where F_n, F_i are "almost" dyadic partitions.

Let us denote $\overline{w}_k(f,t)_p$ as the sum of all partial modules of continuity of a function f. There exists a connection between $\overline{w}_k(f,t)_p$ and piecewise-polynomial approximations.

Theorem 2. *Let $f \in L_p(Q_0), 0 < p \leq \infty$. There is a way to select special "almost" dyadic partitions $\widetilde{F}_n^i$, such that*

$$\overline{w}_k(f, 2^{-n})_p \approx \sum_{i=1}^{2^d} e_k(f, \widetilde{F}_n^i)_p.$$

Using Theorem 1 and Theorem 2 the following corollary can be obtained:

Corollary 1. *Let $f \in L_p(Q_0)$, $0 < p < q < \infty$, then*

$$\overline{\omega}_k(f, 2^{-n})_q \leq c \cdot \left(\sum_{i=n}^{\infty} \left(2^{id\left(\frac{1}{p}-\frac{1}{q}\right)} \overline{\omega}_k(f, 2^{-i})_p \right)^q \right)^{\frac{1}{q}}.$$

IMAGE QUALITY ASSESSMENT AND WAVELETS

V.V. Khryashchev , D.A. Zaramensky
Yaroslavl, Russia
connect@piclab.ru

Historically full-reference image quality assessment methods used simple mathematic evaluations such as peak signal-to-noise ratio (PSNR). These methods were fast and simple but not well correlated with subjective criterion mean opinion score (MOS) and were not universal.

In modern image processing application modified image quality index (MIQI) is used. This criterion is based on wavelet decomposition and on human visual system peculiarities. The MIQI algorithm consists of applying universal image quality index (UIQ) [1] to each subband in one level wavelet decomposition of original and given images. The resulted MIQI is calculated as:
$MIQI = 0.57 \cdot UIQ_{LL} + 0.17 \cdot UIQ_{LH} + 0.15 \cdot UIQ_{HL} + 0.09 \cdot UIQ_{HH}$
were weights corresponds to human visual sensitivity to the subbands.

This algorithm was tested during the visual experiment for calculating MOS for more than 100 images with different type of distortions (JPEG, JPEG2000, impulse noise, additive white Gaussian noise (AWGN)). The results are shown in table 1.

Таблица 1: Correlation between MOS and objective estimations.

Distortion	Objective Estimations		
	PSNR	UIQ	MIQI
Gaussian Blur	0.396	0.733	0.732
JPEG	0.517	0.776	0.890
JPEG2000	0.845	0.751	0.878
Impulse Noise	0.950	0.944	0.884
AWGN	0.979	0.893	0.936

MIQI distinguishes images equal in PSNR metric and shows good correlation with MOS. This criterion will be helpful in developing new optimal image processing algorithms and optimizing existing ones. The common drawback of UIQ and MIQI algorithms is that original image is needed.

References

1. Wang Z., Bovik A. A universal image quality index // IEEE Signal processing letters. 2002. V. 9, № 3. P. 81-84.

REPRESENTATIONS OF ALMOST PERIODIC FUNCTIONS USING WAVELET FRAMES

Yeonhyang Kim, Amos Ron
Central Michigan University and University of Wisconsin-Madison
kim4y@cmich.edu

We represent the space of almost periodic (AP) functions using a *wavelet* system. With this representation in hand, our objective is to compute, or at least to estimate, the norm of the underlying function. Our observation is that this norm estimation of AP functions is valid if and only if the given *wavelet* system is an $L_2(\mathbb{R})$-frame. Moreover, the frame bounds of the system are also the sharpest bounds in our estimation. This gives a surprising connection between $L_2(\mathbb{R}^d)$-representations and AP-representations.

References

1. F. Galindo, *Some remarks on: "On the windowed Fourier transform and wavelet transform of almost periodic functions"*, Appl. Comput. Harmonic Anal., **16(3)**, 174-181, 2004.

2. Y. Kim, A. Ron, *Time frequency representations of almost periodic functions*, Constr. Approx., (DOI: 10.1007/s00365-007-9000-0), 2008

3. A. Ron, Z. Shen, *Frames and stable bases for shift-invariant subspaces of* $L_2(\mathbb{R}^d)$, Canad. J. Math., **47(5)**, 1051-1094, 1995.

4. A. Ron, Z. Shen, *Affine systems in* $L_2(\mathbb{R}^d)$*: the analysis of the analysis operator*, J. Funct. Anal., **148**, 408-447, 1997

SMOOTH FUNCTIONS ASSOCIATED WITH WAVELET SETS ON $\mathbb{R}^d$, $d \geq 1$, AND FRAME BOUND GAPS

Emily King
UMD, USA
eking@math.umd.edu

The theme is to smooth characteristic functions of Parseval frame wavelet sets by convolution in order to obtain implementable, computationally viable, smooth wavelet frames. We introduce the following: a new method to improve frame bound estimation; a shrinking technique to construct frames; and a nascent theory concerning frame bound gaps. The phenomenon of a *frame bound gap* occurs when certain sequences of functions, converging in L^2 to a Parseval frame wavelet, generate systems with frame bounds that are uniformly bounded away from 1. We prove that smoothing a Parseval frame wavelet set wavelet on the frequency domain by convolution with elements of an approximate identity produces a frame bound gap. Furthermore, the frame bound gap for such frame wavelets in $L^2(\mathbb{R}^d)$ increases and converges as d increases.

WAVELET DENOISING OF EXPERIMENTAL DATA WITH NON-STATIONARY NOISE

T.N. Knyazeva, N.I. Oreshko
Research and Engineering Center of Saint Petersburg
Electrotechnical University, St. Petersburg, Russia
ktn@nic.spb.ru, oreshko@nic.spb.ru

Numerous existing methods of denoising which have already been introduced in regression and wavelet analyses work effectively only for stationary noise or on the assumption that the noise model may be considered non-stationary and noise itself can be approximated by a smooth curve with preliminary known parameters. However, according to the results obtained after analyzing a wide range of processes encountered in various practical applications, the original signal often includes both the wanted quasideterministic signal and the non-stationary noise with the variance that might change at random time points and has an unknown general law. In the present paper some new contributions to the field of signal denoising are proposed. They are all based on the wavelet transform and are intended to process signals embedded in non-stationary noise. These new methods including some specific features of the tasks dealing with non-stationary noise are expounded in further context.

In many well-known denoising techniques the whole original signal is used for calculating special thresholds which are applied to contaminated signals in order to eliminate noise and extract only the pure signal component as a result. The estimation procedure for estimating threshold values involves determining the root-mean-square error (RMSE) calculated on the basis of all signal samples. Nonetheless, sometimes such an approach may lead to unsatisfactory results, for example, when noise variance for some signal fragments differs significantly from that obtained for the rest of a signal and, therefore, the local RMSE is not equal to the global one.

The first method is aimed at removing noise whose variance changes rapidly at some time points (hence, there is a jump change), i.e. the corresponding variance dependence in time is a piecewise-constant function. In accordance with the new denoising strategy, which has been developed and thoroughly investigated, two subsequent steps are needed.

At first, cluster-analysis (directed at splitting the original data into a number of groups with similar properties) is employed with the purpose of classifying all wavelet coefficients that are located at decomposition levels. Thus, we will manage to discover the groups of data possessing constant variance, the groups indicating some typical signal fragments. Then threshold calculation for each cluster discovered is done separately so that the values themselves are different from each other unlike those provided after the widely used classical thresholding techniques where thresholds are calculated either only once (the single-level thresholding) or for each level independently (the multilevel thresholding) without performing any wavelet coefficient classification.

The main idea behind the second step is to find the law to which RMSE change of noise is generally subject. Even though this law is not known in advance, it should be definitely represented by a smooth curve; otherwise this strategy will not succeed or produce inaccurate results in the end. Due to the fact that the wavelet transform is interpreted as a dyadic filter bank structure, there is no difficulty extracting noise. The algorithm consists of four main steps: 1) The magnitudes of the noise component derived are defined; 2) The mean trend in noise is defined; 3) The original signal is normalized by means of being divided by the trend that was previously estimated. It is also necessary to take into account a normalizing factor caused by the law of half-normal distribution; 4) Threshold processing of wavelet coefficients by means of the threshold selected is carried out. Finally, via the inverse wavelet transform the pure original signal is reconstructed.

The new methods offered in the manuscript extend the possibilities of noise removal with the help of the underlying thresholding strategy. Furthermore, they allow one to cope with heteroscedastic noise, which is of paramount importance here in view of incapability of most classical methods to deal with this problem.

References

1. Donoho D., Johnstone I. Adapting to unknown smoothness via wavelet shrinkage// J. Amer. Statist. Assoc. 1995. V.90. P.1200-1224.

2. Inclan C., Tiao G. Use of Cumulative Sums of squares for

retrospective defection of change of variance// JASA. 1994. V.427. P.913-935.

3. Knyazeva T. N., Novikov L. V., Oreshko N. I. Removal of Non-stationary Noise from Experimental Data// St.Petersburg: Scientific Instrument Making, 2008. V.18 (N.2). P.61-65.

4. Percival D., Walden A. Wavelet methods for time series analysis. London: Cambridge University Press, 2000. 594 p.

5. Rawlings J., Pantula S., Dickey D. Applied Regression Analysis. Spinger 1989. 658 p.

6. Strang G., Nguyen T. Wavelets and Filter Banks. Boston: Wellesley-Cambridge Press, 1996. 512 p.

7. Vidakovic B. Statistical Modeling by Wavelets. New York: John Wiley & Sons, 1999. 382 p.

SOME EXTREMAL PROBLEMS OF VARIOUS METRICS AND SHARP INEQUALITIES OF NAGY-KOLMOGOROV TYPE

V. Kofanov
Dnepropetrovsk, Ukraine
vladimir.kofanov@gmail.com

For $r \in \mathbf{N}$ and $p \in (0, \infty]$ we denote by $L^r_{p,\infty}(\mathbf{R})$ the space of all functions $x : \mathbf{R} \to \mathbf{R}$ for which $x^{(r-1)}$ are locally absolutely continuous, $x \in L_p(\mathbf{R})$ and $x^{(r)} \in L_\infty(\mathbf{R})$.

For any fixed interval $[\alpha, \beta] \subset \mathbf{R}$, given $r \in \mathbf{N}$, and γ, δ, $p > 0$, we solve the following extremal problems

$$\left\| x^{(k)} \right\|_{W_q} \to \sup \quad \text{and} \quad \left\| x^{(k)} \right\|_{L_q[\alpha,\beta]} \to \sup$$

over all functions $x \in L^r_{p,\infty}(\mathbf{R})$ such that $L(x)_p \le \gamma$, $\left\| x^{(r)} \right\|_\infty \le \delta$, in the cases 1) $k = 0$, $q \ge p$, 2) $1 \le k \le r-1$, $q \ge 1$, where

$$\|x\|_{W_q} := \lim_{\Delta \to \infty} \sup_{a \in \mathbf{R}} \left(\frac{1}{\Delta} \int_a^{a+\Delta} |x\,(t)|^q \, dt \right)^{1/q},$$

$$L(x)_p := \sup\left\{\left(\int\limits_a^b |x(t)|^p dt\right)^{\frac{1}{p}} \;:\; a,b \in \mathbf{R},\ |x(t)|>0,\ t\in(a,b)\right\}.$$

We prove also the following sharp inequality of Nagy-Kolmogorov type

$$\left\|x^{(k)}\right\|_{W_q} \le \frac{\|\varphi_{r-k}\|_{W_q}}{L(\varphi_r)_p^\alpha} L(x)_p^\alpha \left\|x^{(r)}\right\|_\infty^{1-\alpha}, \quad x\in L_{p,\infty}^r(\mathbf{R}),$$

in the cases 1) and 2), where φ_r is the perfect Euler's spline of order r and $\alpha=(r-k)/(r+1/p)$.

Besides, we show that for any ω, γ, $\delta>0$ there exists a function $x\in L_{p,\infty}^r(\mathbf{R})$ such that

$$\left\|x^{(k)}\right\|_{W_q} = \omega, \quad L(x)_p=\gamma, \quad \left\|x^{(r)}\right\|_\infty = \delta$$

if and only if

$$\omega \le \frac{\|\varphi_{r-k}\|_{W_q}}{L(\varphi_r)_p^\alpha}\,\gamma^\alpha\,\delta^{1-\alpha}.$$

The generalization of Calderon and Klein's inequality for the entire functions of exponential type σ is established.

ON APPROXIMATION OF FUNCTIONS BY TRIGONOMETRIC POLYNOMIALS WITH INCOMPLETE SPECTRUM IN $L_p, 0<p<1$

Yurii S. Kolomoitsev
Institute of Applied Mathematics and Mechanics, Donetsk, Ukraine
kolomus1@mail.ru

Let A be a proper subset of $\mathbb{Z}$. Then the system $\{e^{ikx}\}_{k\in A}$ is not complete in the space $L_p(0,2\pi)$ for $p\ge 1$. A somewhat different situation arises in L_p with $p<1$.

For some sets $A\subset\mathbb{Z}$ that possess certain arithmetic properties, the estimates of the best approximation

$$E_n(f,A)_p := \inf\{\|f-T\|_{L_p(0,2\pi)} : T\in span\{e^{ikx}\}_{k\in A\cap(-n,n)}\}$$

are obtained.

Consider the class of functions

$$H^{\alpha}_{1,p} := \{\, f \;:\; \|f\|_1 + \sup_{n\geq 1} n^{\alpha} E_{n-2}(f,\mathbb{Z})_p \leq 1\}.$$

Theorem *Let* $0 < p < 1$, $Q = \mathbb{Z} \setminus \{\pm q^k\}_{k\in\mathbb{N}}$ $(q \geq 2)$ *and* $n \in \mathbb{N}$. *The following statements hold:*

(i) if $0 < \alpha < \frac{1}{p} - 1$, *then*

$$\sup_{f\in H^{\alpha}_{1,p}} E_n(f,Q)_p \asymp n^{-\alpha};$$

(ii) if $\frac{1}{p} - 1 \leq \alpha \leq \frac{2}{p} - 2$, *then*

$$C_1 n^{1-\frac{1}{p}} \leq \sup_{f\in H^{\alpha}_{1,p}} E_n(f,Q)_p \leq C_2(\ln(n+1))^{\frac{1}{p}} n^{1-\frac{1}{p}};$$

(iii) if $\alpha > \frac{2}{p} - 2$, *then*

$$\sup_{f\in H^{\alpha}_{1,p}} E_n(f,Q)_p \asymp n^{1-\frac{1}{p}}$$

where $\asymp$ *is two-sided inequalities with positive constants that depend only on* p, q *and* α; C_1 *and* C_2 *are the positive constants that depend only on* p *and* q.

Similar results are obtained for the sets $S = \mathbb{Z} \setminus \{\pm k^s\}_{k\in\mathbb{N}}$ $(s \geq 2)$ and $M = \mathbb{Z} \setminus (-m, m)$.

FRAMES OF p-ADIC WAVELETS AND REPRESENTATION THEORY

S.V. Kozyrev
Steklov Mathematical Institute, Moscow, Russia
kozyrev@mi.ras.ru

Development of the p-adic wavelet theory was initiated in [1].

We discuss the new approach to the p-adic wavelet theory based on the representation theory of p-adic groups. In this approach frames of wavelets are considered as orbits (i.e. systems of coherent states, cf. [2]) of some groups of transformations.

It is easy to see that the action of a sufficiently small transformation (i.e. belonging to some vicinity of the unit of the group of transformations) on a p-adic test function (locally constant function with compact support) leaves this function invariant.

Therefore the orbit of a test function will be a discrete set. We show that this set in some interesting cases can be computed explicitly and will be (for the cases under consideration) a frame of wavelets.

We discusss the following two cases.

1) In the one dimensional case [3] we consider the action of the p-adic affine group on a generic mean zero locally constant function with compact support (mean zero test function). We show that this orbit is a uniform tight frame and compute the bound of this frame.

We discuss the relations of this result with the multiresolution wavelet analysis and show that the introduced approach allows to construct multiresolution frames as well as frames which can not be described by the multiresolution construction.

2) We construct a multidimensional basis of p-adic wavelets [4] using the direct generalization of the one dimensional case of [1]. The relation of the constructed basis to representations of the p-adic group of transformations, generated by translations, homogeneous dilations and norm conserving linear transformations, is discussed. We show that the set of products of the vectors from the constructed basis and p-roots of one is the orbit of this p-adic group of transformations.

We also show that this multidimensional wavelet basis coincides with the basis generated by the multiresolution construction in many dimensions (which for $p = 2$ was discussed in [5]).

Keywords: p–adic wavelets, frames of wavelets, multiresolution analysis, representation theory

References

1. Kozyrev S.V., Wavelet theory as p-adic spectral analysis // Izvestiya: Mathematics, 2002, **66** (2), 367–376. arXiv:math-ph/0012019

2. Perelomov A., Generalized Coherent States and Their Applications. Berlin, Springer, 1986.

3. Albeverio S., Kozyrev S.V., Frames of p-adic wavelets and orbits of the affine group // p-Adic Numbers, Ultrametric Analysis and Applications, 2009, **1** (1), 18–33. arXiv:0801.4713

4. Albeverio S., Kozyrev S.V., Multidimensional basis of p-adic wavelets and representation theory // arXiv:0903.0461

5. Shelkovich V.M., Skopina M.A., p-Adic Haar multiresolution analysis // arXiv:0704.0736

PARAMETRIZATION OF BIVARIATE NONSEPARABLE HAAR WAVELETS. [2]

M.S. Krasilnikova

Voronezh State University, Russia

krasilnikovams@nm.ru

A parametrization of all orthogonal wavelet bases for Haar multiresolution analysis is derived. The bases generated by three piecewise constant wavelets $\{\eta_i(x,y)\}$ supported on $[0,1]\times[0,1]$ with values a_{ij}, $i=1,2,3; j=1,2,3,4$ are considered.

In order to describe the values a_{ij} geometrical approach was used. After two changes of variables we have the following system of equations:

$$\begin{cases} \cos(\beta_1-\beta_2) = -4\operatorname{ctg}\alpha_1\operatorname{ctg}\alpha_2 \\ \cos(\beta_2-\beta_3) = -4\operatorname{ctg}\alpha_2\operatorname{ctg}\alpha_3 \\ \cos(\beta_1-\beta_3) = -4\operatorname{ctg}\alpha_1\operatorname{ctg}\alpha_3 \end{cases}$$

Where $\alpha_1,\alpha_2,\alpha_3$ must satisfy the following condition:

$$1-16\operatorname{ctg}^2\alpha_1\operatorname{ctg}^2\alpha_2-16\operatorname{ctg}^2\alpha_1\operatorname{ctg}^2\alpha_3-16\operatorname{ctg}^2\alpha_2\operatorname{ctg}^2\alpha_3- \\ -128\operatorname{ctg}^2\alpha_1\operatorname{ctg}^2\alpha_2\operatorname{ctg}^2\alpha_3=0$$

Choosing α_2,α_3 as independent variables we solve this system in regard to β_2,β_3 (β_1 is assumed to be equal to 1 for simplicity). The signs of arc cosines are dependent on the given α_2,α_3. The inverse changes of variables will give the required description of the values.

[2]This work is supported by RFBR (grant 08-01-00226-a).

The cases when some of the values are equal to zero are of interest in application. In this work different variants of zeroes' arrangement are studied and the general type of such systems is described.

References

1. Novikov I.Ya., Protasov V. Yu., Skopina M.A. Theory of Wavelet. - M.: Physmatlit, 2005. - 616p.
2. K. Blutter. Wavelet analysis. The fundamentals of the theory. - M.: Tehnosfera, 2006.-272p.

DIFFERENT EXTENSIONS OF THE WIENER'S TAUBERIAN LEMMA AND MEMORY LOCALIZATION.

Ilya A. Krishtal
Northern Illinois University, USA
krishtal@math.niu.edu

In this semi-expository talk I will describe a unified point of view on how different extensions of the Wiener's Tauberian Lemma are related to different types of memory localization such as off-diagonal decay, frame localization, etc. Localized LU-type factorizations and connections with sampling theory will be mentioned.

MULTIVARIATE FRAME-SIMILAR WAVELET SYSTEMS

Alexander Krivoshein.
St. Petersburg State University
Department of Applied Mathematics and Control Processes
St. Petersburg, Russia.
san_san@inbox.ru

Let M be an integer $d \times d$ matrix whose eigenvalues are bigger than 1 in module, $m = |\det M|$; if $f \in L_2(\mathbb{R}^d)$, then $f_{jk} := m^{j/2} f(M^j \cdot +k)$.

A general scheme for the construction of dual wavelet frames is well known [1]. One starts with a pair of refinable masks $m_0, \widetilde{m}_0$ and finds wavelet masks $m_\nu, \widetilde{m}_\nu$, $\nu = 1, \dots, r$, $r \geq m - 1$, according to Unitary Extension Principle [1]. The corresponding wavelet functions

$\psi^{(\nu)}, \widetilde{\psi}^{(\nu)}$ generate dual wavelet systems $\{\psi_{ik}^{(\nu)}\}$, $\{\widetilde{\psi}_{ik}^{(\nu)}\}$. The systems form dual frames whenever $\psi^{(\nu)}, \widetilde{\psi}^{(\nu)}$ satisfy a number of conditions. Unfortunately, it is hard enough to check these conditions in practice (in fact, some smoothness of $\psi^{(\nu)}, \widetilde{\psi}^{(\nu)}$ should be checked). If we have only $\psi^{(\nu)}, \widetilde{\psi}^{(\nu)} \in L_2(\mathbb{R}^d)$, then the wavelet systems $\{\psi_{jk}^{(\nu)}\}$, $\{\widetilde{\psi}_{jk}^{(\nu)}\}$ is said to be **dual frame-similar wavelet systems**.

Theorem 1. *Let* $\{\psi_{ik}^{(\nu)}\}$ $\{\widetilde{\psi}_{ik}^{(\nu)}\}$ *be dual frame-similar compactly supported wavelet systems. Then for all* $f, g \in L_2(\mathbb{R}^d)$ *we have*

$$\langle f, g\rangle = \sum_{i=-\infty}^{\infty} \sum_{\nu=1}^{r} \sum_{k\in\mathbb{Z}^d} \langle f, \widetilde{\psi}_{ik}^{(\nu)}\rangle \langle \psi_{ik}^{(\nu)}, g\rangle.$$

If $n \in \mathbb{N}$, $n \geq 1$, *is the order of vanishing moments of the wavelet functions* $\widetilde{\psi}^{(\nu)}(x)$, *then for all* f *from the Sobolev space* W_2^n *we have*

$$\left\| f - \sum_{i=-\infty}^{j-1} \sum_{\nu=1}^{r} \sum_{k\in\mathbb{Z}^d} \langle f, \widetilde{\psi}_{ik}^{(\nu)}\rangle \psi_{ik}^{(\nu)} \right\|_2 \leq C \frac{\|f\|_{W_2^n}}{(|\lambda| - \epsilon)^{jn}},$$

where λ *is a minimal (in module) eigenvalue of* M, $\epsilon > 0$, $|\lambda| - \epsilon > 1$, C *is a constant which does not depend on* f, j, *i.e. the frame-similar decomposition converges in norm and has approximation order* n.

The case $d = 1$, $M = 2$ was investigated with M.Skopina in [2].

References

1. Ron A. and Shen Z., *Affine systems in* $\mathbb{L}_2(R^d)$*: the analysis of the analysis operator* // J. Func. Anal. – 1997 – **148** – P. 408–447.

2. Кривошкин А.В., Скопина М.А., *Фреймоподобные системы всплесков* // Зб. праць Ін-ту математики НАН України. — 2009. — Т. 6, №1. — С. 96–114

$\mathcal{AD}$-FRAMES SATISFYING PROPERTY $\mathcal{B}$

Shiv Kaushik Kumar [1], **Varinder Kumar**[2]
[1]Department of Mathematics, Kirori Mal College (University of Delhi), Delhi, INDIA
shikk2003@yahoo.co.in
[2]Department of Mathematics, University of Delhi, Delhi, INDIA
vicky.h1729@gmail.com

A generalization of atomic decompositions for Banach spaces, namely $\mathcal{AD}$-frames has been introduced and studied. A characterization of $\mathcal{AD}$-frames satisfying property $\mathcal{B}$ has been given. Also, we gave a sufficient condition for an $\mathcal{AD}$-frame to satisfy property $\mathcal{B}$ and a necessary condition for a particular type of $\mathcal{AD}$-frame satisfying property $\mathcal{B}$. Finally, we prove a result regarding quasi-reflexivity of Banach spaces having $\mathcal{AD}$-frames satisfying property $\mathcal{B}$ and property $\mathcal{RSM}$.

2000 Mathematics Subject Classification. 42C15, 42A38, 42C40

Key Words: Atomic decompositions; Frames; $\mathcal{AD}$-frames.

FRAMES AND THE KADISON-SINGER PROBLEM

W. Lawton
Department of Mathematics National University of Singapore
Singapore
matwml@nus.edu.sg

Let $\phi =$ Fourier transform of the characteristic function of a Cantor set with positive measure. Bownik and Speegle [2] ask if $Z = \Lambda_1 \cup \cdots \cup \Lambda_n$ with each $\Lambda_j(\phi) = \{ \phi(\cdot - \tau) : \tau \in \Lambda_j \}$ a Riesz basis? Since a yes answer requires a Λ_j with positive density, they asserted ([2], page 1142) that the result of Bourgain and Tzafriri [1] showing the existence of such a set "remains the strongest indicator that the answer to" their question is yes. We prove that the answer to their question is yes iff there exists a syndetic Λ and study properties of $\Lambda(\phi)$, for some sets Λ arising in harmonic analysis [6] and ergodic theory [4], that suggest a no answer to their question and hence by [3] to the Kadison-Singer problem [5].

References

1. J. Bourgain and L. Tzafriri, *Invertibility of "large"submatrices with applications to the geometry of Banach spaces and harmonic analysis*, Israel Journal of Mathematics, (2) **57** (1987), 137-224.
2. M. Bownik and M. Speegle, *The Feichtinger conjecture for wavelet frames, Gabor frames and frames of translates*, Canad. J. Math, (6) **58** (2006), 1121-1143.
3. P. Casazza, O. Christensen, A. Lindner, and R. Vershynin, *Frames and the Feichtinger conjecture*, Proc. Amer. Math. Soc., **133** (2005), 1025-1033.
4. H. Furstenberg, *Recurrence in Ergodic Theory and Combinatorial Number Theory*, Princeton Univ. Press, New Jersey, 1981.
5. R. Kadison and I. Singer, *Extensions of pure states*, Amer. J. Math., **81**, (1959), 547-564.
6. Y. Meyer, *Algebraic Numbers and Harmonic Analysis*, North-Holland, Amsterdam, 1972.

QUASISPLINE WAVELET [3]

E.A. Lebedeva

Kursk State University, Russia

ealebedeva2004@gmail.com

The article [1] shows that the uncertainty constants tend to infinity as smoothness grows for a broad class of orthogonal scaling functions and wavelets, for instance Daubechies wavelets and Battle-Lemarie wavelets. However, a family of modified Daubechies wavelets is described in [2] and [3]; the time-frequency localization of the autocorrelation function constructed for the scaling function of a wavelet of this family is preserved with the growth of smoothness. New scaling functions and wavelets, introducing in [4], decay exponentially at infinity and have the decay of order $O(\omega^l)$ as $|\omega| \to \infty$ in the frequency domain, like spline

[3]This work is supported by RFBR (grant 09-01-00162).

wavelets; the uncertainty constants of the scaling functions (wavelets) tend to those of the Meyer scaling function (the Meyer wavelet) with respect to the smoothness parameter l.

The function $m_l(\omega) := (\cos 0,5\omega)^{2l} V_{N(l)}(m_l^M, \omega) \left(V_{N(l)}(m_l^M, 0)\right)^{-1}$, is the mask of the stable, but not orthogonal scaling function φ_l : $\widehat{\varphi}_l(\omega) := \prod_{j=1}^{\infty} m_l\left(\frac{\omega}{2^j}\right)$. By $V_{N(l)}(m_l^M, \omega)$ denote the de la Vallee-Poussin mean of m_l^M, where $m_l^M(\omega) := m^M(\omega)(\cos 0,5\omega)^{-2l}$, and m^M is a mask of the Meyer wavelet. The orthonormal scaling function $\varphi_l^{\perp}$ is defined by $\widehat{\varphi_l^{\perp}}(\omega) := \widehat{\varphi}_l(\omega) \left(\sum_{k\in\mathbb{Z}} |\widehat{\varphi}_l(\omega + 2\pi k)|^2\right)^{-0,5}$.

Application of some others linear methods of summability instead of the de la Vallee-Poussin means is discussed.

References

1. Chui C.K., Wang J. High-order orthonormal scaling functions and wavelets give poor time-frequency localization // The J. of Fourier Anal. and Appl., Vol. 2, No. 5, 1996, pp. 415-426.

2. Novikov I. Ya. Modified Daubechies wavelets preserving localization with growth of smoothness // East J. Approximation, Vol. 1, No. 3, 1995, pp. 314-348.

3. Novikov I. Ya.. Uncertainly constants for modified Daubechies wavelets [in Russian] // Izv. Tul'sk. Gos. Univ. Ser. Mat. Mekh. Inform., Vol. 4, No. 1, 1998, pp. 107-111.

4. Lebedeva E.A. Exponentially decaying wavelets with uncertainty constants uniformly bounded with respect to the smoothness parameter // Sib. Math. J., Vol. 49, No. 3, 2008, pp. 457-473.

APPROXIMATION OF SOBOLEV AND OTHER CLASSES BY POLYNOMIALS AND RIDGE FUNCTIONS

D. Leviatan
TAU, Israel
leviatan@math.tau.ac.il

Joint work with V. N. Konovalov and V. E. Maiorov

Let $W_p^r(\mathbb{B}^d)$ be the usual Sobolev class of functions on the unit ball $\mathbb{B}^d$ in $\mathbb{R}^d$, and $W_p^{\circ,r}(\mathbb{B}^d)$ be the subclass of all radial functions in $W_p^r(\mathbb{B}^d)$. We show that for the classes $W_p^{\circ,r}(\mathbb{B}^d)$ and $W_p^r(\mathbb{B}^d)$, the orders of best approximation by polynomials in $L_q(\mathbb{B}^d)$ coincide. We also obtain exact orders of best approximation in $L_2(\mathbb{B}^d)$ of the classes $W_p^{\circ,r}(\mathbb{B}^d)$ by ridge functions and, as an immediate consequence, we obtain the same orders in $L_2(\mathbb{B}^d)$ for the usual Sobolev classes $W_p^r(\mathbb{B}^d)$.

We also obtain estimates on the order of best approximation by polynomials and ridge functions in the spaces L_q of classes of s-monotone radial functions which belong to the space L_p, $1 \le q \le p \le \infty$.

HAAR SERIES ON LOCALLY COMPACT ZERO-DIMENSIONAL ABELIAN GROUP

S.F. Lukomskii
Department of Mathematics, Saratov University, Saratov, Russia
lukomskiisf@info.sgu.ru

Let (G, f) be a locally compact periodic abelian group and a topology in group $(G, \dot{+})$ given by a chain of subgroups

$$\cdots \supset G_{-n} \supset \cdots \supset G_{-1} \supset G_0 \supset G_1 \supset \cdots \supset G_n \supset \dots$$

with $G = \bigcup_{n\in\mathbf{Z}} G_n$, $\{0\} = \bigcap_{n\in\mathbf{Z}} G_n$. Let p_n be the order of factor-group G_n/G_{n+1}, p_n – are prime numbers. We set $m_0 = 1$, $m_{n+1} = m_n \cdot p_n$. Let X denote the group of characters of the group G, $G_n^\perp = \{\chi \in X : \chi(G_n) = 1\}$ – annulators of $G_n^\perp$. The set $(G_n^\perp)_{n=-\infty}^{+\infty}$ of annulators is the increasing sequence

$$\cdots \subset G_{-n}^\perp \subset \cdots \subset G_{-1}^\perp \subset G_0^\perp \subset G_1^\perp \subset \cdots \subset G_n^\perp \subset \dots$$

and $(G_{n+1}^\perp/G_n^\perp)^\sharp = p_n$. Picking up one element $g_n \in G_n \setminus G_{n+1}$ and one character $r_n \in G_{n+1}^\perp \setminus G_n^\perp$ for each $n \in \mathbf{Z}$. We define Haar-functions $H_{jm_n+k}(x)$ as $H_{jm_n+k}(x) = \sqrt{m_n} r_n^j(x \dot{-} q)\mathbf{1}_{G_n \dot{+} q}(x) \quad (j =$

$\overline{1, p_n - 1}; n \in \mathbf{Z})$ where k and q are connected with next condition $q = g_{n-1}\alpha_{n-1} \dot{+} g_{n-2}\alpha_{n-2} \dot{+} \ldots \dot{+} g_{n-s}\alpha_{n-s} \Leftrightarrow$
$\Leftrightarrow k = m_{n-1}\alpha_{n-1} \dot{+} m_{n-2}\alpha_{n-2} \dot{+} \ldots \dot{+} m_{n-s}\alpha_{n-s}$.

Theorem 1. *The system* $(H_{jm_n+k})_{n\in\mathbf{Z}}$ *is orthonormal system on* G *complete in* $L_2(G)$.

Theorem 2. *Let* $f: G \to \mathbf{C}$ *be continuous on* G *and* $\omega_n(f)$ *be a modulus of continuity of* f. *If* $\lim\limits_{n\to+\infty} p_n\omega_n(t) \to 0$, *then the Haar-Fourier series of* f *converge uniformly on any subgroup* G_N $(N \leq 0)$.

If $p_n = p$ for any $n \in \mathbf{Z}$, then we write Haar-function in the form $H_{jp^n+k}(x) = p^{\frac{n}{2}} r_0^j(A^n(x \dot{-} q))\mathbf{1}_{G_n \dot{+} q}(x)$ $(n \in \mathbf{Z})$, where A is dilation operator for which $A(G_n) = G_{n-1}$, $A(x \dot{+} y) = Ax \dot{+} Ay$. In these case we can obtain Haar-functions H_{jp^n+k} from one function $r_0(x)\mathbf{1}_{G_0}(x)$ using raising to a power, dilation and translation. From one system $\{r_0(x), r_0(x)^2, \ldots, r_0(x)^{p-1}\}$ we can obtain $2^{\frac{p-1}{2}}$ different real-valued systems $\left\{\psi_1^{(l)}(x), \psi_2^{(l)}(x), \ldots, \psi_{p-1}^{(l)}(x)\right\}$ $(l = 1, \ldots, 2^{\frac{p-1}{2}})$.

This work supported by President program (grant 2970.2008.1)

References

1. Benedetto J.I., Benedetto R.L. A wavelet theory for local field and related groups// The J. of Geometric Analysis , v.14, N3(2004), p.423-456.

DECOMPOSITION FORMULAS FOR INSERTED GROUP OF KNOTS

A.A. Makarov
St. Petersburg State University, Russia
Antony.Makarov@gmail.com

Normalized B_φ-splines of the second order are constructed. The splines are continuously differentiable and have the minimal compact support. Arbitrary grid refinement by successive inserting of single knots or group of knots is regarded. Representation of splines for initial grid as a linear combination of splines for refined grid is done. Embedding of spaces of B_φ-splines for irregular grids is established. This leads to a wavelet decomposition (e. g. signals with fast oscillations). The wavelet basis has compacted support. The decomposition and reconstruction formulas are obtained.

EQUIANGULAR TIGHT FRAMES

Vasily N. Malozemov[†], **Alexander B. Pevnyi**[‡]
[†]St. Petersburg State University, Russia
[‡]Syktyvkar State University, Russia
[†]*malv@math.spbu.ru,* [‡]*pevnyi@syktsu.ru*

This talk is based on the review paper [1].

Four equivalent definitions of a tight frame are discussed. A construction of a known Mercedes-Benz frame is generalized for a case of an n-dimensional space. Notions of Mercedes-Benz systems and Mercedes-Benz matrix are introduced. A question of existence of equiangular tight frames is studied.

Extremal properties of tight frames, Mercedes-Benz systems and equiangular tight frames are pointed out.

References

1. V. N. Malozemov, A. B. Pevnyi, "Equiangular tight frames", J. Math. Sciences, vol. 157, no. 6, pp. 789–815, 2009.

PARAMETRIC FAST FOURIER TRANSFORM AND WAVELET EXPANSIONS

Vasily N. Malozemov[†], **Oleg V. Prosekov**[‡]
[†]St. Petersburg State University, Russia
[‡]ARC International, St. Petersburg, Russia
[†]*malv@math.spbu.ru,* [‡]*Oleg.Prosekov@ARC.com*

The general approach to constructing FFTs involves the decomposition of the Fourier matrix into a product of sparse matrices. Various versions of this decomposition depend on the arithmetic properties of the order of the Fourier matrix and on representations of its indices. In this talk we present the parametric coding of indices and obtain the "perfect" parametric decomposition of the Fourier matrix.

Let $N = n_1 n_2 \cdots n_s$, $\Delta_\nu = n_1 n_2 \cdots n_{\nu-1}$ (for $\nu \in 2 : s+1$, $\Delta_1 := 1$) and $N_\nu = n_{\nu+1} n_{\nu+2} \cdots n_s$ (for $\nu \in 0 : s-1$, $N_s := 1$).

Theorem. For any parameter vector $p = (p_1, p_2, \dots, p_s)$, the Fourier matrix F_N admits the representation

$$F_N = \big(Rev^{(q_1,\dots,q_s)}_{n_1,\dots,n_s}\big)^T \Bigg(\prod_{\nu=1}^{s} \big(I_{N_\nu} \otimes Twid^{(p_1,\dots,p_{\nu-1},q_\nu)}_{n_1,\dots,n_{\nu-1},n_\nu}\big) \times$$

$$\times \, \big(I_{N_\nu} \otimes F_{n_\nu} \otimes I_{\Delta_\nu}\big) \Bigg) Mix^{(p_1,\dots,p_s)}_{n_1,\dots,n_s} \,,$$

where $\otimes$ denotes Kronecker multiplication of matrices, I_N is the identity matrix of order N and $q = (q_1, q_2, \dots, q_s)$ is the adjoint parameter vector with elements are defined by the condition $\langle q_\nu \, p_\nu \rangle_{n_\nu} = 1$ for $\nu \in 1 : s$.

Also this decomposition involves special matrices: permutation matrices Rev, Mix and diagonal matrix $Twid$. The parameter vector can be chosen so that the number of nontrivial elements (different from ± 1 and $\pm i$) in the parametric twiddle matrix decreases in comparison with the usual (nonparametric) twiddle matrix. Incidentally, permutation matrices have its decompositions.

Basing on this factorization, we construct sequences of orthogonal bases in a signal space which generate a parametric wavelet packet.

References

1. Malozemov V. N., Prosekov O. V. *Parametric Versions of the Fast Fourier Transform* // Doklady Mathematics. 2008. Vol. 78. No. 1. P. 576–578.

MULTIVARIATE COMPLEX B-SPLINES, DIRICHLET AVERAGES AND APPROXIMATION

Peter Massopust
Institute of Biomathematics and Biometry
Helmholtz Zentrum München
and
Centre of Mathematics, M6
Technische Universität München
Germany
massopust@ma.tum.de

Complex B-splines B_z are an extension of cardinal B-splines to complex orders $\Re z > 1$. It was shown in [**FBU**] that such complex B-splines generate a multiresolution analysis of $L^2(\mathbb{R})$. In [**FM1,FM2**] several connections between complex B-splines, Dirichlet averages, fractional derivative and integral operators were exhibited and an extension to the multivariate setting was given. Several properties of multivariate complex B-splines were presented in [**FM3,M**].

In this talk, a summary of the properties of multivariate complex B-splines, their connection to Dirichlet averages, fractional derivative and integral operators, as well as complex difference operators is given. Some new results are also presented. Finally, the approximation-theoretic properties of multivariate complex B-splines are discussed and the connection to multiresolution analyses exhibited.

References

FBU B. Forster, T. Blu, and M. Unser, “Complex B-splines,"Appl. Comp. Harmon. Anal. **20** (2006), 261–282.

FM1 B. Forster and P. Massopust, “Statistical Encounters with complex B-splines,"to appear in Constr. Approx.

FM2 P. Massopust and B. Forster, “Multivariate complex B-splines and Dirichlet averages,"to appear in J. Approx. Th.

FM3 B. Forster and P. Massopust, “Multivariate complex B-splines, Dirichlet averages and difference operators submitted to SAMPTA 2009

M P. Massopust, “Double Dirichlet Averages and Complex B-Splines,"sumitted to SAMPTA 2009.

THE COMPARATIVE CHARACTERISTIC OF POLYNOMIAL AND SPLINE METHODS OF APPROXIMATION OF FUNCTIONS OF ONE VARIABLE

Valery Miroshnichenko
Sobolev Institute of Mathematics, Novosibirsk, Russia
miroshn@math.nsc.ru

The methods comparison of polynomial and spline [2] approximation regarding the accuracy of approximation of smooth enough single-variable functions is given in the talk.

- We give an information on the known accuracy estimates of approximations of function by polynomials and splines.
- We give the numerical results on accuracy of approximation of some functions, such as $f(x) = (1 + x^2)^{-1}$ (example of Runge [1]) and other “externally harmless” functions. At the same time the recommendations for stable computation of polynomial approximations are given.

Adduced results show that even “not the best” interpolating polynomial splines regarding the accuracy of approximation exceed any polynomial methods of approximation, including the best uniform and the best mean-square approximation. Thereupon no wonder that known attempts of perfection of polynomial methods of approximation inevitably lead to a piecewise polynomial approximations, i.e. essentially to the splines.

References

1. *Runge C.* Über empirische Funktionen und die Interpolation zwischen äquidistanten Ordinaten // Zeitschrift für Mathematik und Physik. 1901. B. 46. S. 224–243.
2. *Zavjalov Yu. S., Kvasov B. I., Miroshnichenko V. L.* Methods of spline functions. Moskow: Nauka. 1980. (Russian)

ON UPPER AND LOWER RIESZ BOUNDARIES FOR B-SPLINE BASES

E.V. Mishchenko
Sobolev Institute of Mathematics, Novosibirsk, Russia
eugenia-m@academ.org

Exact expressions for upper and lower Riesz boundaries for B-spline bases of arbitrary orders are found. The obtained results make possible studying the convergency of power series of certain type and uniform convergency of the Battle-Lemarier scaling functions in the space $L^2(\mathbf{R})$.

Let $\mathbf{A_m}, \mathbf{B_m}$ denote the lower and upper Riesz boundaries for B-spline basis of order m, i.e.

$$2\pi \mathbf{A_m} \cdot \sum_{k=-\infty}^{\infty} |c_k|^2 \le || \sum_{k=-\infty}^{\infty} c_k B_m(\cdot - k)||^2_{L^2(R)} \le 2\pi \mathbf{B_m} \cdot \sum_{k=-\infty}^{\infty} |c_k|^2$$

for every $\{c_k\} \in l_2$.

Theorem 1.

$$\mathbf{A_m} = \frac{2^{2m+2}}{\pi^{2m+3}} \sum_{k=0}^{\infty} \frac{1}{(1+2k)^{2(m+1)}}, \quad \mathbf{B_m} = \frac{1}{2\pi}.$$

Theorem 2. Let $\psi_m(x) = \phi_m(x - \frac{\varepsilon}{2})$, here ϕ_m is the Battle-Lemarier function of the order m, $\varepsilon = 0$, if m is even; $\varepsilon = 1$ if m is odd. Then $||\psi_m - \phi||_{L^2(\mathbf{R})} \to 0$ as $m \to m$, where ϕ is the Shennon-Kotel'nikov function.

WINDOWED EXPONENTIALS IN FUNCTION SPACES [4]

S.Ya. Novikov
Akad.Pavlov, Dept.of Math., Samara St.Univ., Samara, Russia
nvks@ssu.samara.ru

Windowed exponentials are defined in function spaces on bounded sets. Let $\mathbf{B}$ be a bounded set in $\mathbf{R}^d$, $g \in L^2(\mathbf{B})$, Λ be a sequence of points in $\mathbf{R}^d$. The sequence

$$\mathcal{E}(g, \Lambda) = \left\{e^{2\pi i \lambda \cdot x} g(x)\right\}_{\lambda \in \Lambda}$$

[4]supported by the Ministry of Education and Science of the Russian Federation, the analytical departmental target program "Development of scientific potential of the higher school"grant N 3341

is called a *system of windowed exponentials* in $L^2(\mathbf{B})$.

For $g(x) \equiv 1$, $\mathcal{E}(g, \Lambda)$ is the classical system of exponentials [1].

The connection between the properties of the system of windowed exponentials in $L^2(\mathbf{B})$ and Beurling density of Λ was investigated in [2]. We consider similar connections in other Function Spaces such as weigthed L^p_w and modulation spaces.

References

1. Sedletcii A.M *Classes of analytic Fourier transforms and exponential approximations.* — M.,Fizmatlit, 2005.

2. Heil Ch.,Kutyniok G. *Density of Frames and Schauder Bases of windowed exponentials.* Houston J. of Math. - 2008. - V.34, No. 2. P. 565-599.

SAMPLING OF BAND-LIMITED SIGNALS.

A.M. Olevskii
TAU, Israel
olevskii@yahoo.com

How often one should measure a "signal" with a given spectrum in order to be able to recover it? What is the optimal "sampling"? Is it possible to organize it so that it will work for any spectrum of given size, independently on its localization?

I will survey classical background and discuss some recent results (joint with A.Ulanovskii).

ON AN OPERATOR-VALUED T(1) THEOREM FOR VECTOR-VALUED QUASI-LIPSCHITZ SPACES

B.I. Peleshenko
Dnepropetrovsk State Agrarian University, Ukraine
dsaupelesh@mail.ru

In this paper we prove the operator-valued T(1) theorem for vector-valued spaces defined by means of local approximations.

ℓ^1 GREEDY ALGORITHM FOR SPARSE SOLUTIONS OF UNDERDETERMINED LINEAR SYSTEMS.

A. Petukhov, I. Kozlov
UGA, USA

The algorithm combining best properties of Orthogonal Greedy Algorithm and ℓ^1 minimization algorithm and outperforming both of them will be presented.

Its applications to information theory (compressed sensing) and image compression will be discussed.

USING OF THE CONTOURLET TRANSFORM FOR THE DIGITAL IMAGE FILTRATION

A. Priorov, V. Volokhov
Yaroslavl State University, Yaroslavl, Russia
volokhov@piclab.ru

Currently, there are many different methods to restore noisy signals and images. Images are 2D-signals with the inherent geometric structure, which is the main feature of the visual information. Features are located along a smooth curve - contours, due to smooth surfaces of section of displayed objects. The use of separable or inseparable wavelet transform does not perform the processing of the images in the form of curved contours, boundaries, etc. In this paper we describe an algorithm for image reconstruction from noisy data based on contourlet transform [1]. The new transform has the following properties:

1. Multiresolution. The representation allow images to be successively approximated, from coarse to fine resolutions.

2. Localization. The basis elements in the representation localized in both the spatial and the frequency domains.

3. Critical sampling. The representation has form a basis, or a frame with small redundancy.

4. Directionality. The representation contains basis elements oriented at a variety of directions, much more than the few directions that are offered by separable wavelets.

5. Anisotropy. To capture smooth contours in images [2], the representation contains basis elements using a variety of elongated shapes with different aspect ratios.

Algorithm of image filtering is as follows:

1. Computing of the contourlet transform images.

2. Changing of the coefficients of contourlet transform for a particular rule. In this paper we used an algorithm based on a hard threshold processing transform coefficients.

3. Computing of the inverse contourlet transform.

References

1. Do M.N., Vetterli M. The contourlet transform: an efficient directional multiresolution image representation // IEEE Trans. Image Proc. 2005. V.14, № 12. P. 1-16.

2. Bamberger R. H., Smith M. J. T. A filter bank for the directional decomposition of images: theory and design // IEEE Trans. Signal Proc. 1992. V.40, № 4. P. 882-893.

WAVELETS AND EQUATIONS OF SELF-SIMILARITY

Vladimir Protasov

Moscow State University, Russia

v-protassov@yandex.ru

Wavelet-functions ψ generating compactly supported wavelets, and the corresponding refinable functions φ can be considered as special cases of the following general construction. Let $L_p[0,1]$ be the space of vector-functions from the segment $[0,1]$ to $\mathbf{R}^d$ with the norm $\|v\|_p = \bigl(\int_0^1 |v(t)|^p\,dt\bigr)^{1/p}$, let also $\{\tilde{A}_1, \ldots, \tilde{A}_m\}$ be a family of irreducible affine operators in $\mathbf{R}^d$, and $0 = b_0 < \ldots < b_m = 1$ be a partition of the segment $[0,1]$. The *self-similarity operator* $\tilde{\mathbf{A}}$:

$$\bigl[\,\tilde{\mathbf{A}}\,v\,\bigr](t) \;=\; \tilde{A}_k\,v\bigl(g_k^{-1}(t)\bigr), \qquad t \in \Delta_k, \qquad k = 1, \ldots, m,$$

is defined on $L_p[0,1]$, where we denote $\Delta_k = [b_{k-1}, b_k]$ and the affine function $g_k(t) = t\,b_k + (1-t)\,b_{k-1}$ maps $[0,1]$ to the segment Δ_k. The equation $\tilde{\mathbf{A}}v = v$ is called *self-similarity equation.* For any system of compactly-supported wavelets both ψ and φ can be obtained as solutions of suitable self-similarity equations. Moreover, most of the classical fractal curves (such as Cantor singular function, Koch and de Rham curve, etc.) are solutions of this equation. Equations of this type appear naturally in the ergodic theory (self-similar measures), probability (distributions of power random series), etc.

We present a sharp criterion of solvability of this equation in terms of the affine operators $\tilde{A}_j$ and of the segments Δ_j in $L_p[0,1]$. It appears that the solution is always unique, whenever it exists. Moreover, the iterations of the operator $\tilde{\mathbf{A}}$ always converge to this solution for any initial function. The exponents of local and global regularity of the solution $v(t)$ and its moduli of continuity can be expressed by spectral characteristics of the operator $\tilde{\mathbf{A}}$.

Applying these results to wavelets theory gives the estimations for moduli of continuity of compactly supported wavelets in the spaces C and L_p, and explicit formulae for their local regularity at a given point.

References

1. V. Protasov, *Fractal curves and wavelets*, Izvestiya Mathematics, **70** (2006), No 5, 105-145.

2. V. Protasov, *Extremal L_p-norms and self-similar functions*, Linear Algebra Appl., **428** (2008), 2339-2356.

ОБ ОДНОМ НЕРАВЕНСТВЕ ДЛЯ ЧИСЛОВЫХ РЯДОВ

Е.И. Радзиевская
IMATH, Ukraine
radz@imath.ua

Пусть $\xi := \{\xi_j\}_{j\in N}$ последовательность вещественных чисел, у которых $\xi_j \geq \xi_{j+1}$, $j \in N$ и $\lim_{j\to\infty} \xi_j = 0$. Каждой такой $\xi = \{\xi_j\}_{j\in N}$ сопоставим последовательность $\xi^* = \{\xi_j^*\}_{j\in N}$, полагая $\xi_j^* = |\xi_{\varphi(j)}|$, где $\varphi(.)$ — такая перестановка натурального ряда, что $\{|\xi_{\varphi(j)}|\}_{j\in N}$ является невозрастающей последовательностью. Для двух последовательностей $\alpha = \{\alpha_j\}_{j\in N}$ и $\xi = \{\xi_j\}_{j\in N}$ определим их произведение $\alpha\xi := \{\alpha_j\xi_j\}_{j\in N}$, а если все элементы $\alpha_j \neq 0$, то $\alpha^{-1} := \{\alpha_j^{-1}\}_{j\in N}$ и $\xi/\alpha := \xi\alpha^{-1}$.

Введем банахово пространство $\mathbf{l}_r$, $1 \le r < \infty$, состоящее из последовательностей $\xi = \{\xi_j\}_{j \in N}$, удовлетворяющих условию

$$\|\xi\|_r := \left(\sum_{j=1}^{\infty} |\xi_j|^r \right)^{1/r} < \infty, \quad \xi \in \mathbf{l}_r.$$

Пусть далее $1 < p < \infty$, $q := p(p-1)^{-1}$, а $\{k_s\}_{s \in N}$ некоторая подпоследовательность натуральных чисел.

Рассмотрим последлвательность $\omega = \{\omega_s\}_{s \in N}$ с элементами

$$\omega_s := \left(\sum_{j=k_s}^{k_{s+1}-1} (\alpha_j^*)^{-p} \right)^{-1/p} \left(\sum_{j=k_s}^{k_{s+1}-1} \nu_j \right).$$

где $\alpha_j > 0$, $0 \le \nu_j \le \nu_{j+1}$ для всех $j \in N$ и $\alpha\nu \in \mathbf{l}_q$.

И, наконец, через Γ обозначим множество всех перестановок натурального ряда, а для $\gamma := \{\gamma(j)\}_{j \in N}$ из Γ и последовательности $\xi = \{\xi_j\}_{j \in N}$ положим $(\xi)_\gamma := \{\xi_{\gamma(j)}\}_{j \in N}$.

Во введенных обозначениях справедливы следующие утверждения

Теорема. *Пусть* $1 \le p \le \infty$, $\lim_{j \to \infty} \alpha_j = 0$ *и* $\sup_{j \in N} \alpha_j \nu_j < \infty$. *Тогда*

$$\inf_{\gamma \in \Gamma} \|\nu(\alpha\eta)_\gamma\|_p \le \omega \|\eta\|_1, \quad \eta \in \mathbf{l}_1,$$

причем постоянную

$$\omega := \sup_{j \in N} \frac{((\nu_1)^p + \ldots + (\nu_k)^p)^{1/p}}{(\alpha_1^*)^{-1} + \ldots + (\alpha_k^*)^{-1}}$$

в этом неравенстве нельзя уменьшить.

ON 1959 KADISON-SINGER CONJECTURE AND FRAMES

Oleg Reinov [5]
Department of Mathematics, Saint Petersburg State University
St. Petersburg, Russia
orein51@mail.ru

We are going to discuss, from different points of view, the famous 1959 Kadison-Singer Conjecture (KS): every pure state on the C^*-algebra D of bounded diagonal operators on the Hilbert space l_2 has a unique extension to a (pure) state on $B(l_2)$, the C^*-algebra of all bounded linear operators on l_2. This problem arose from the very productive collaboration of Kadison and Singer in the 1950s which culminated in their seminal work on triangular operator algebras. During this collaboration, they often discussed the fundamental work of Dirac — P.A.M. Dirac, Quantum Mechanics, 3rd Ed., Oxford University Press, London (1947). In particular, they kept returning to one part of Dirac's work because it seemed to be problematic.

Dirac wanted to find a "representation"(an orthonormal basis) for a compatible family of observables (a commutative family of self-adjoint operators). Dirac's claim, in mathematical form, is that each pure state of a "complete commuting set"has a unique state extension to $B(l_2)$. By a "complete"commuting set, Dirac means what is now called a "maximal abelian self-adjoint"subalgebra of $B(l_2)$; D is one such. There are others. For example, another is generated by an observable whose "simple"spectrum is a closed interval. Kadison and Singer show that that is not so for each complete commuting set other than D. They also show that each pure state of D has a unique extension to the uniform closure of the algebra of linear combinations of operators T defined by $T_\pi e_i = e_{\pi(i)}$, where π is a permutation of Z, $\{e_i\}$ is the orthonormal basis of l_2. Kadison and Singer believed that KS had a negative answer. In particular, on page 397 of the paper "R. Kadison and I. Singer, Extensions of pure states, American Jour. Math. 81 (1959), 383-400"they state: "We incline to the view that such extension is non-unique".

We will consider KS-problem within the framework of Frame Theory for Hilbert spaces, introducing, in particular, some new notion

[5] This research was done with partial support by the Fond RFFI Grant 06-01-00457

connected with frames.

PROPERTIES AP_p AND AP^p IN BANACH SPACES

Oleg Reinov [6]
Department of Mathematics, Saint Petersburg State University
St. Petersburg, Russia
orein51@mail.ru

The natural generalizations of AP and BAP (the properties AP_p, BAP_p, AP_p^{dual} etc.) were considered earlier by P. Saphar, the author and others. All these properties were firstly defined in terms of some tensor products (generalizations of Grothendieck ones). One of the question of P. Saphar was to describe the AP_p in terms of something like "compact convergence"for absolutely p-summing operators. It was done earlier by the author of this talk. This small lecture is devoted, mainly, to the proceeding in the same direction for the class (ideal) Π_p^d of dually absolutely-p-summing operators, and giving the connections between some notions of "compact convergence"of type π_p^{dual} and the properties of tensor products, with applications. We will apply the results to the investigation of a natural question: when a Banach space has the generalized approximation properties (such as introduced by the author the properties AP^p)? We give here some sufficient and some necessary conditions for the space to have AP^p as well as construct some (counter)examples to the AP^p-approximation problems.

In the very end of the talk, we apply our results to give a new proof of a much more stronger result than one from my paper in C. R. Acad. Sc. Paris (concerning a conjecture of A. Grothendieck from his fundamental work on tensor products). Answering in negative to the Grothendieck question whether every weakly compact operator with the AP has also the BAP, we have constructed earlier the example of a compact operator with AP, but without BAP. Here, in Theorem, we show that there exist the operators of such a kind, belonging even to the classes of dually quasi-p-nuclear (hence, compact) operators.

[6]This research was done with partial support by the Fond RFFI Grant 06-01-00457

ON CHOOSING A SMOOTHING PARAMETER WITH HIGH-ORDER CONVERGENT ALGORITHM

Alexander I. Rozhenko
Institute of Computational Mathematics and Mathematical Geophysics
Siberian Branch of Russian Academy of Sciences
Novosibirsk, Russia
rozhenko@oapmg.sscc.ru

The abstract smoothing spline is defined as the solution of minimization problem

$$\sigma_\alpha := \arg\min_{x\in X} \{\alpha\|Tx\|^2 + \|Ax - z\|^2\},$$

where X, Y, and Z are real Hilbert spaces, $A : X \to Z$ and $T : X \to Y$ are bounded linear operators, $z \in Z$ is a data vector, and $\alpha > 0$ is a smoothing parameter.

Given an estimate ε for a noise, the standard choice of the parameter α is that σ_α should satisfy the residual equation

$$\varphi(\alpha) := \|A\sigma_\alpha - z\| = \varepsilon. \tag{1}$$

In the standard algorithms [1, 2], the residual equation is transformed to the equation $1/\varphi(\beta) = 1/\varepsilon$ with $\beta := 1/\alpha$, which is then solved with Newton's method.

Based on Tailor expansions of the residual operator $R_\alpha z := z - A\sigma_\alpha$ and of its complementary operator $Q_\beta = I - R_{1/\beta}$ about given α_0 and $\beta_0 := 1/\alpha_0$, respectively, [3], we obtain a family of two-sided estimates for the values of $\|R_\alpha z\|$ and $\|R_{1/\beta}z\|$ and introduce a new method for solving the equation (1). In comparison with Newton's method which converges quadratically, the convergence rate of our method can be of any given order.

References

1. Reinsch, C.H., Smoothing by spline functions. II, Numer. Math., 1971, 16, 451–454.

2. Gordonova, V.I. and Morozov, V.A., Numerical algorithms of parameter selection in regularization method, Zh. Vychisl. Mat. i Mat. Fiz., 1973, 13, 539–545 (in Russian).

3. Rozhenko, A.I., On optimal choice of spline-smoothing parameter, Bull. of Novosibirsk Computing Center, Series Num. Anal., NCC Publisher, Novosibirsk, 1996, 7, 79–86.

DYADIC MULTIWAVELET TRANSFORMS [7]

Pavel G. Severov
Voronezh State University, Russia
severovpg@gmail.com

For Alpert spline-multiwavelets of order r=2,3 the following equality is valid:

$$\sum_{k=1}^{r} |\widehat{\varphi^{(k)}}(\xi)|^2 + |\widehat{\psi^{(k)}}(\xi)|^2 = \sum_{k=1}^{r} |\widehat{\varphi^{(k)}}(\xi/2)|^2,\ \xi \in R.$$

As a consequence we have the following decomposition of unity:

$$\sum_{k} \sum_{l=1}^{r} |\widehat{\varphi^{(l)}}(2^k \omega)|^2 \equiv 1.$$

References

1. A. J. Stacey, W. F. Blyth, A variational method using Alpert multiwavelets, ANZIAM J.48 (CTAC2006) pp. C820-836,2008. Available at http://anziamj.austms.org.au/ojs/index.php/ANZIAMJ/article/view/73

2. Fritz Keinert, Wavelet and Multiwavelet, Chapman & Hall/CRC, 2004.

3. Novikov I.Ya., Protasov V. Yu., Skopina M.A. Theory of Wavelet. - M.: Physmatlit, 2005. - 616p.

[7] This work is supported by RFBR (grant 08-01-00226-a).

p-ADIC WAVELETS, PSEUDO-DIFFERENTIAL OPERATORS AND EQUATIONS

V.M. Shelkovich
St.-Petersburg State Architecture and Civil Engineering University
St. Petersburg, Russia
shelkv@yahoo.com

We introduce a notion of *p-adic multiresolution analysis* (*MRA*) and using the equation $\phi(x) = \sum_{r=0}^{p-1} \phi\big(\frac{1}{p}x - \frac{r}{p}\big)$, $x \in \mathbf{Q_p}$, as the "natural" *refinement equation* we study a concrete MRA being an analog of Haar MRA in $L^2(\mathbf{R})$ [4]. In contrast to the Haar MRA in $L^2(\mathbf{R})$, there exist *infinity many different* p-adic orthonormal wavelet bases in $L^2(\mathbf{Q_p})$ generated by the same Haar MRA [3], [4]. In [1], it was proved that there are no other orthogonal MRA based wavelet bases. We also study p-adic *refinement equations* and their solutions [2]. One of them coincides with the above "natural" refinement equation. A wide class of p-adic *refinable functions* generating a MRA is described. It was proved in [1] that there exist no orthogonal test refinable functions different from those described in [2] and all these functions generate the same p-adic Haar MRA. We also construct the Haar multidimensional wavelet bases by means of a tensor product of one-dimensional MRAs.

Next, we study some problems related with the theory of multidimensional p-adic wavelets in connection with the theory of multidimensional p-adic pseudo-differential operators and pseudo-differential equations.

The author was supported by DFG Project 436 RUS 113/809 and DFG Project 436 RUS 113/951 and by Grant 05-01-04002-NNIOa of Russian Foundation for Basic Research.

References

1. S. Albeverio, S. Evdokimov, M. Skopina, *p-Adic multiresolution analysis*, (2008), http://arxiv.org/abs/0810.1147v1

2. A.Yu. Khrennikov, V.M. Shelkovich, M. Skopina, p-Adic refinable functions and MRA-based wavelets, to appear in Journal of Approximation Theory, (2009).

3. A.Yu. Khrennikov, V.M. Shelkovich, M. Skopina, p-Adic orthogonal

wavelet bases, to appear in p-Adic Numbers, Ultrametric Analysis and Applications, (2009).

4. V. M. Shelkovich, M. Skopina, p-Adic Haar multiresolution analysis and pseudo-differential operators, to appear in Journal of Fourier Analysis and Applications, (2009).

POLYPHASE METHOD FOR THE CONSTRUCTION OF MULTIVARIATE WAVELET FRAMES

M. Skopina
St. Petersburg State University
Department of Applied Mathematics and Control Processes
St. Petersburg, Russia
skopina@MS1167.spb.edu

Our polyphase method allows to construct a compactly supported tight wavelet frame with a given approximation order starting with any appropriate refinable mask. A general form for all appropriate initial refinable masks is described by an explicit formula. An insignificant modification of this method for the matrix dilations whose determinant is odd leads to wavelet frames generated by symmetric/untisymmetric wavelet functions. A method for the construction of dual wavelet frames is also discussed.

TIGHT FRAMES OF SPECIAL FORM

Natalia A. Solovjova
Saint-Petersburg State University
The Faculty of Mathematics and Mechanics
St. Petersburg, Russia
4vinyo@gmail.com

Suppose U is a unitary $(n \times n)$–matrix with eigenvalues $\lambda_1, \ldots, \lambda_n$ and corresponding orthonormal eigenvectors $p_1, \ldots, p_n$. Let us fix a unit-norm vector $\varphi_0 \in \mathbb{C}^n$ and assuming that $m \geq n$ build the vector system

$$\left\{\varphi_0, U\varphi_0, U^2\varphi_0, \ldots, U^{m-1}\varphi_0\right\}. \tag{1}$$

Theorem 1. *The system* (1) *is a tight frame if and only if two following conditions hold:*

1. *the eigenvalues* $\lambda_1, \ldots, \lambda_n$ *of the matrix* U *are distinct* m*-roots of a certain number* $c \in \mathbb{C}$, $|c| = 1$;
2. $|\langle \varphi_0, p_k \rangle| = \frac{1}{\sqrt{n}}$ *for all* $k \in 1 : n$.

Let us examine the vectors

$$\eta_k = P^* \varphi_k, \qquad k \in 0 : m-1,$$

where P is a unitary matrix with the columns $p_1, \ldots, p_n$.

Theorem 2. *The system* $\{\eta_0, \eta_1, \ldots, \eta_{m-1}\}$ *is a general harmonic frame if and only if conditions* (1) *and* (2) *of the theorem 1 hold.*

As an application of theorem 2 it is shown how to reduce the Mercedes-Benz frame to a general harmonic frame.

References

1. P. G. Casazza and J. Kovačević, «Equal-norm tight frames with erasures», *Adv. Comp. Math.* (*Special Issue Frames*), vol. 18, no. 2–4, pp. 387–430, 2002.

BEST m-TERM APPROXIMATIONS OF THE CLASSES $B^r_{p,\theta}$ OF PERIODIC FUNCTIONS BY POLINOMIALS BY HAAR SYSTEM

S.A. Stasyuk
Institute of Mathematics NASU, Kyiv, Ukraine
stasyuk@imath.kiev.ua

Let R^d, $d \geq 1$, be the d-dimensional space of elements $x = (x_1, \ldots, x_d)$, and let $L_p([0,1]^d)$ be the space of all 1-periodic functions f with norm

$$||f||_p = \left(\int_{[0,1]^d} |f(x)|^p \, dx \right)^{1/p}, \quad 1 \leq p < \infty,$$

and with natural modification for $p = \infty$.

For given set $\mathcal{D}$ of elements of some Banach space B the best m-term approximation of element $f \in B$ with regard to system $\mathcal{D}$ is

$$\sigma_m(f, \mathcal{D})_B = \inf_{g_i \in \mathcal{D}, c_j} \left\| f - \sum_{j=1}^{m} c_j g_j \right\|_B .$$

For classes $B^r_{p,\theta} \subset L_q$ let $\sigma_m(B^r_{p,\theta}, \mathcal{D})_q = \sup_{f \in B^r_{p,\theta}} \sigma_m(f, \mathcal{D})_{L_q}$.

Theorem. *If* $1 < p \leq \infty$, $1 < q < \infty$, $1 \leq \theta \leq \infty$, $\frac{d}{p} < r < 1$ *and* $\mathcal{H} = \{H_I\}_I$ *is Haar system, then*

$$\sigma_m(B^r_{p,\theta}, \mathcal{H})_q \asymp m^{-r}.$$

For Nikol'skii classes $H^r_p = B^r_{p,\infty}$ result of theorem coinside in [1], when $d = 1$. Let us remark also that the quantities $\sigma_m(B^r_{p,\theta}, \mathcal{T})_q$ in the case of trigonometrical system $\mathcal{T} = \{e^{i(k,x)}\}_k$ were investigated in [2]. If we compare the theorem with corresponding estimates of quantities $\sigma_m(B^r_{p,\theta}, \mathcal{T})_q$ from [2], we can see that the approximations on Haar system are superior to the approximations on trigonometrical system in some relations between the parameters p and q.

References

1. Andrianov A.V. *Approximation of functions belonging to the classes MH^r_q by Haar polynomials.* (Russian), Mat. Zametki, **66**(3)(1999), P. 323–335.

2. De Vore R.A., Temlyakov V.N. *Nonlinear approximation by trigonometric sums,* The Journal of Fourier Analysis and Applications, **2**(1)(1995), P. 29–48.

PECULIARITIES OF MULTIDIMENSIONAL WAVELET FUNCTIONS AND MULTIRATE SYSTEMS SYNTHESIS

Mikhail K. Tchobanou

Moscow Power Engineering Institute (Technical University)

Moscow, Russia

cmk2@orc.ru

The task of biorthogonal and orthogonal multidimensional wavelet functions and multirate systems design is considered. The analytical design methods for multidimensional nonseparable signals and multirate systems under investigation were developed earlier by the author. The conditions for multiresolution analysis construction are investigated, an example for multidimensional Cohen condition checking is provided for one of design methods.

References

1. I. Novikov, V. Protasov and M. Skopina. *Wavelet theory.* FIZMATLIT, M., 2005.
2. M. Tchobanou. Synthesis of basic elements of multidimensional multirate systems. Part. 1. Nonseparable decimation matrices. *Numerical Analysis and Applications*, 2008, 1(1):79–94.
3. M. Tchobanou. Design of basic elements of multidimensional multirate systems. Multidimensional filters with fractional shift. *Numerical Analysis and Applications*, 2008, 2(1):179–195.
4. M. Tchobanou. Synthesis of multidimensional filter banks with even support. *Computational Technologies*, Novosibirsk, 2008, 13(5):95–112.
5. M. Tchobanou. Hierarchical coding algorithm for nonseparable lattices and filter banks. *Computational Technologies*, Novosibirsk, 2007, 12(4):106–119.
6. M. Tchobanou. Even supported multidimensional filter banks. *Proc. of The 2007 International Workshop on Spectral Methods and Multirate Signal Processing, SMMSP. Moscow.* (37):45–50, 2007.
7. M. Tchobanou and S. Karakazian. Three-channel multirate systems *Bulletin of Saint-Petersburg University, Series 10: Applied Mathematics, informatics, control processes*, (1):66–75, 2007.

FRAMES IN BANACH SPACES

P. A. Terekhin
N. G. Chernyshevskii Saratov State University, Russia
terekhinpa@info.sgu.ru

Let X be the Banach space of sequences satisfying the following condition: the system of unit vectors forms a basis in the space X. This condition allows us to identify the dual space X^* with some Banach space of sequences Y. We say that a system of nonzero vectors $\{\varphi_n\}_{n=1}^{\infty}$ forms a *frame in a Banach space F with respect to the space of sequences*

X if the Fourier coefficients of an arbitrary continuous linear functional $g \in G = F^*$ satisfy the inequalities

$$A\|g\|_G \leq \|\{\langle \varphi_n, g\rangle\}_{n=1}^{\infty}\|_Y \leq B\|g\|_G,$$

where $0 < A \leq B < \infty$ are constants not depending on g. Any frame is a repsentation system and any repsentation system is a frame with respect to the its coefficient space. However, a representation system may form frames with different space of sequences.

Theorem 1. *Let $\{\varphi_n\}_{n=1}^{\infty}$ be a frame in a Banach space F with respect to the space of sequences X. Then the following conditions are equivalent:*

- *the frame $\{\varphi_n\}_{n=1}^{\infty}$ is a projection of a basis of an ambient space, i.e., there exist a Banach space $\mathcal{F} \supset F$ with basis $\{e_n\}_{n=1}^{\infty}$ whose coefficient space coincides with X and projection $P : \mathcal{F} \to F$ such that $\varphi_n = Pe_n$, $n = 1, 2, \ldots$;*
- *there exists a system $\{\psi_n\}_{n=1}^{\infty} \subset F^*$ such that, for any $f \in F$, $\{\langle f, \varphi_n\rangle\}_{n=1}^{\infty} \in X$ and $f = \sum_{n=1}^{\infty}\langle f, \psi_n\rangle\varphi_n$;*
- *the coefficient space of zero-series $N = \{x : \sum_{n=1}^{\infty} x_n\varphi_n = 0\}$ is complemented in X.*

This research was supported by the program “Leading Scientific Schools” under grant no. 2970.2008.1.

References

1. P. A. Terekhin, “Representation systems and projections of bases”, Mat. Zametki [Math. Notes], **75** (2004), no. 6, 944-947.

STEERABLE WAVELET FRAMES IN $L_2(R^D)$

Michael Unser
Biomedical Imaging Group EPFL, CH-1015 Lausanne, Switzerland
Michael.Unser@epfl.ch

We introduce an Nth-order extension of the Riesz transform in d dimensions. We prove that this generalized transform has the following remarkable properties: shift-invariance, scale-invariance, inner- product preservation, and steerability. The pleasing consequence is that the transform maps any primary wavelet frame (or basis) of $L_2(R^d)$ into another "steerable"wavelet frame, while preserving the frame bounds. Concretely, this means we can design reversible multi-scale decompositions in which the analysis wavelets (feature detectors) can be spatially rotated in any direction via a suitable linear combination of wavelet coefficients. The concept provides a rigorous functional counterpart to Simoncelli's steerable pyramid whose construction was entirely based on digital filter design. The proposed mechanism allows for the specification of wavelets with any order of steerability in any number of dimensions; it also yields a perfect reconstruction filterbank algorithm. We illustrate the method using a Laplacian-like (or Mexican hat) polyharmonic spline wavelet transform as our primary frame. We display new wavelets that replicate the behavior of the Nth-order partial derivatives of an isotropic Gaussian kernel.

This is joint work with Dimitri Van De Ville.

CUBIC SHAPE PRESERVING SPLINE INTERPOLATION

Yuri Volkov
Sobolev Institute of Mathematics, Novosibirsk, Russia
volkov@math.nsc.ru

In the talk we discuss the classical interpolation problem by C^2 cubic splines. In spite of the fact that cubic splines are considered as the basic and universal tool in the majority of problems connected with practical approximation of functions, they are not ideal, and in some problems it is necessary to refuse them. Shape preserving interpolation, i.e. approximation with the conservation of geometrical properties of data such as positivity, monotonicity, convexity is among these problems. Certainly, numerous modifications and generalizations of cubic splines have been developed for such cases. However, here we almost always loose some attractive properties of a classical spline such as smoothness, accuracy, approximation order, simplicity of realization and so on. Thus the problem of finding conditions of shape preserving for traditional cubic spline interpolation is very actual.

We propose simple conditions for the cubic spline to inherit geometrical properties of the initial function. Namely, if the interpolated function is k-monotone (the k-th divided differences of data are positive), then these conditions will guarantee that the usual cubic interpolation spline is k-monotone, i.e. its k-th derivative is positive. We examine $k = 0, 1, 2, 3, 4$.

Our approach uses idea of [1], and is based on a new representation of interpolation splines [2], [3].

This work was supported by the grant of the Department of Mathematical Sciences, Russian Academy of Sciences (project 2009-3.8), the Integration Projects grant, Siberian Division of the Russian Academy of Sciences (project 2009-81), and the Joint Integration Projects grant, Siberian and Ural Divisions of the Russian Academy of Sciences (project 2009-14).

References

1. *Miroshnichenko V. L.* Convex and monotone spline interpolation // Constructive theory of function'84. Sofia, 1984, 610–620.

2. *Volkov Yu. S.* A new method for constructing cubic interpolating splines // Comp. Math. & Math. Phys. 2004. 44, 2, 215–224.

3. *Volkov Yu. S.* Totally positive matrices in the methods of constructing interpolation splines of odd degree // Sib. Adv. Math. 2005. 15, 4, 96–125.

ON A CONSTRUCTION OF WAVELETS WITH COMPACT SUPPORT BY MEANS OF SPLINES

Zygmunt Wronicz
AGH University of Science and Technology
Faculty of Applied Mathematics
Cracow, Poland
wronicz@agh.edu.pl

In 1982 J.O.Strömberg constructed a wavelet by means of piecewise linear functions, namely a function $\psi \in L^2(\mathbb{R})$, such that the system $\{\psi_{j,k}\}_{j,k\in\mathbb{Z}}$, $\psi_{j,k}(x) = 2^{\frac{j}{2}}\psi\left(2^j x - k\right)$ is an orthonormal basis in the space $L^2(\mathbb{R})$. Unfortunately, the support of this wavelet is not bounded.

In [1] the author constructed a piecewise linear wavelet with support $[0, 2]$.

The talk will be concerned with a construction of spline wavelets with compact support.

References

1. Z.Wronicz, A wavelet with compact support constructed by means of splines, submited to Opuscula Math.

APPLICATION OF WAVELET TRANSFORM TO INHOMOGENEOUS WAVE FIELD ANALYSIS

Li-Chung Wu, Laurence Zsu-Hsin Chuanga, Dong-Jiing Doong, Chia Chuen Kao
National Cheng Kung University, Taiwan (R.O.C.)
jackalson18@gmail.com

The wavelet transform is now recognized as a useful, flexible, and efficient technique to analyze signals and images which are obtained from experiments or in-situ measurements. A new technique based on the two-dimensional wavelet transform (2-D CWT) was developed in this research to represent a wave field locally in both space domain and spatial frequency domain. The required theories of 2-D CWT were collected, defined and explained how its variables related to ocean waves. The relationship between the energy spectra derived by wavelet transform and wavenumber spectra of random waves was then presented in this study. Several numerical simulations were carried out for random ocean wave fields, in which a directional spectrum was assumed, to justify the feasibility of 2-D CWT. The study shows that 2-D CWT is capable of extracting the wavenumber spectrum from a wave field image for any chosen location in the coastal area of varying water depths. And then some more different wave conditions were simulated to verify the accuracy of 2-D CWT in various sea bed slopes. It shows that the accuracy level is sufficient to apply the 2-D CWT to the analysis on ocean and coastal engineering applications.This study also shows a shortcoming in 2-D CWT analysis. The estimated spectral energy and the accuracy of detecting wave parameters are influenced by the distance of location for analyzing from the image edge. The accuracy increases with increasing the distance from the edge. For the sake of obtaining high accurate wave results, the distance between the edge and the calculated location should be larger than half dominate wavelength of the wave field. The comparisons of estimations to theoretical values for several wave parameters show that the continuous wavelet transform is capable of identifying the directional spectra and wave properties in shallow water. After our study, we could conclude that the feasibility of CWT on analyzing the wave image of random waves is palpable, even in the coastal area.

GENERATION OF ENERGY AND CONTENT FREQUENCY COMPATIBLE SIGNALS BASED ON WAVELET TRANSFORM

Azad Yazdani
Department of Engineering, the University of Kurdistan
Sanandaj, Iran
a.yazdani@uok.ac.ir

Wavelet transforms can separate the data into various frequency component, as does the Fourier transform. Unlike the Fourier transform, however, the wavelet transform allows the removal of frequency components at specific times in the data. The discrete wavelet transform (DWT) easily incorporates computed inverse transforms (IDWTs) that allow us to reconstruct the signal after we have identified, removed and changed noisy or superfluous data.

Filtering a signal corresponds to the mathematical operation of the convolutions of the signal with the impulse response to the filter. A half band low-pass filter remove all frequencies that are above half of the highest frequency in the signal. The output give the detail coefficients (from the high-pass filter) and approximation coefficients (from the low-pass filter). The approximate is then further decomposed using the same wavelet decomposition step. While a wavelet analysis involves filtering and down-sampling, the wavelet reconstruction process consists of up-sampling and filtering. To synthesize a signal, up-sampling is the process of lengthening a signal component by inserting zeros between samples.

An iterative procedure has been proposed to modify a signal, here recorded earthquake, so that they become compatible with content frequency and energy of the target signal. The modification has been achieved by decomposing the recorded and target signals through multi-level discrete wavelet transform into wavelet coefficients (cAs, cDs) and by using substitution in matrix process and the inversion operation, IDWT, reconstruct a new signal. In a different iteration step, the sums of the squares of wavelet coefficients are calculated as an indicator of total energy of a signal. In matrix processing, the substituted wavelet coefficient are scaled up/down based on this energy parameter to acquire the target energy.

COIFLETS AND BOUNDARY VALUE PROBLEMS

A.D. Yunakovsky
Institute of Applied Physics RAS, Nizhny Novgorod, Russia
yun@appl.sci-nnov.ru

Several computational techniques for boundary value problems have been developed. The central idea has its origins in the construction of preconditioner in some functional space basis. The coiflet series are very convenient for the approximate calculations since the number of operations for calculating the expansion coefficients as well as the number of operations for reconstruction of the function by means of it's coiflet coefficients, is in proportion with the units in the sample of function.

However, the basis elements of these representations do not satisfy the boundary conditions. This fact leads to a slow convergence of an approximate solution to a precise one.

Convolutions of Green function of model operator satisfying assumed boundary conditions with coiflets have the same compact supports as the corresponding coiflets ψ laying rigorously inside of boundary. Otherwise the convolutions are linear functions outside the support of corresponding coiflets at our disposal intersecting the boundary.

As a result we get a band linear system with diagonal dominance for constructing the approximate solution strongly satysfying the boundary conditions.

GENERALIZATIONS OF THE STRANG–FIX CONDITIONS: CONNECTIONS WITH OPERATOR ADAPTED WAVELETS

Victor G. Zakharov
ICMM, Perm, Russia
victor@icmm.ru

The Strang–Fix conditions are necessary and sufficient conditions for integer shifts of a function f to generate *all* polynomials up to some degree. The Strang–Fix conditions are based on the vanishing of the $\hat{f}$ and its derivatives at a discrete set of points: $\{2\pi n : n \in \mathbb{Z}^N \setminus \{0\}\}$. However we can consider more general set of points. Namely let a function (distribution) $f \in S'(\mathbb{R}^N)$ and $\hat{f}(a) \neq 0$, $a \in \mathbb{C}^N$; then the following conditions are equivalent (see [1,2] for one dimensional case):

(i) $D^\beta \hat{f}(a + 2\pi n) = 0$, $n \in \mathbb{Z}^N \setminus \{0\}$, $|\beta| = 0, \dots, L$;

(ii) $\forall \beta \in \mathbb{Z}^N_{\geq 0}$, $|\beta| \leq L$: $\sum_{k \in \mathbb{Z}^N} p_\beta(k) e^{ia\cdot k} f(x-k) = P_{|\beta|}(x) e^{ia\cdot x}$,

where $\beta = (\beta_1, \dots, \beta_N) \in \mathbb{Z}^N_{\geq 0}$, $|\beta| := \beta_1 + \dots + \beta_N$, $D^\beta := D_1^{\beta_1} \cdots D_N^{\beta_N}$, D_j, $j = 1, \dots, N$, is the partial derivative with respect to the jth coordinate, p_β denotes a monomial $p_\beta(x) := x_1^{\beta_1} \cdots x_N^{\beta_N}$, $x = (x_1, \dots, x_N) \in \mathbb{R}^N$, and $P_r(x)$ is a polynomial of total degree r.

Moreover, in multidimensional case the total degree of the polynomials can be sometimes increased by one. Let $f \in S'(\mathbb{R}^N)$ satisfy the Strang–Fix conditions (item (i) above) and let $\hat{f}(a) = 1$. Fix an arbitrary bijection of the set $\{{}^j\beta : {}^j\beta \in \mathbb{Z}^N_{\geq 0}, |{}^j\beta| = L+1\}$ onto the integers $\{1, \dots, M\}$, $M = \frac{(N+L)!}{(L+1)!(N-1)!}$, and consider vectors $\mathcal{F}(x) := \left(D^{{}^1\beta} \hat{f}(x), \dots, D^{{}^M\beta} \hat{f}(x)\right)$, $\mathcal{P}(x) := \left(p_{{}^1\beta}(x), \dots, p_{{}^M\beta}(x)\right)$. Suppose there exists a vector space $V \subset \mathbb{C}^M$, $V \neq \{(0, \dots, 0)\}$; then $V \perp \mathcal{F}(a + 2\pi n)$, $\forall n \in \mathbb{Z}^N \setminus \{0\}$, iff for any $v \in V$ the following relation holds: $\sum_{k \in \mathbb{Z}^N} v \cdot \mathcal{P}(k) e^{ia\cdot k} f(x-k) = e^{ia\cdot x} \left(v \cdot \mathcal{P}(x) + P_L(x)\right)$.

The generalized Strang–Fix conditions supply necessary and sufficient (in one dimensional case) conditions to obtaining compactly supported wavelets adapted to linear differential operators.

This research is partly supported by the RFBR grant No. 07-01-96048.

References

1. C. Vonesch, T. Blu, M. Unser, *Generalized Daubechies Wavelet Families*, IEEE Transactions on Signal Processing, **55**(9) (2007) 4415–4429.

2. R. Vaillancourt, V.G. Zakharov, *Generalized Strang-Fix conditions: connection with operator adapted wavelets*, Submitted to JWTA, 2008.

ON THE SERIES ARISING AT APPROXIMATION OF PERIODIC, DIFFERENTIABLE FUNCTIONS BY THE POISSON INTEGRALS

V.P. Zastavnyi

Donetsk National University, Donetsk, Ukraine

zastavn@rambler.ru

The following exact value of the remainder under approximation of periodic, differentiable functions by the Poisson Integrals was found in the paper of A.F.Timan in 1950. If $0 \le \rho < 1$ and $r \in N$, then the equality

$$\sup_{f \in W^r} \sup_{x \in R} \left| f(x) - \frac{1}{2\pi} \int_{-\pi}^{\pi} f(x+t) P_\rho(t)\, dt \right| = \\ = \frac{4}{\pi} \sum_{k=0}^{\infty} \frac{(-1)^{k(r+1)}(1-\rho^{2k+1})}{(2k+1)^{r+1}} \quad (1)$$

holds where $P_\rho(t) = \frac{1-\rho^2}{1-2\rho\cos t+\rho^2}$ is the Poisson kernel and a W^r the set of all 2π-periodic functions having absolutely continuous derivatives of the order $(r-1)$ such that $|f^{(r)}(x)| \le 1$ for almost all $x \in R$. For $\rho \to 1-0$ and all $r > 1$ the first $(r-1)$ main terms of an asymptotic expansion of quantity (1) in powers of $\varepsilon = \ln \frac{1}{\rho}$ and integral forms of the truncation errors also are determined in the same paper. The full asymptotic expansion of (1) in powers of $(1-\rho)$ was found by L.V.Malej in 1961 for $r = 1$ (rediscovered by E.L.Stark in 1973) and by K.M.Zhigallo with Yu.I.Harkevich in 2002 for all $r \in N$.

For $r > 1$ all coefficients of this expansion contain both the explicit quantities and quantities which are defined by a complicated recurrence formula. An integral representation of the right side of (1) was found by V.A.Baskakov in 1975 for $r = 1, 2, 3$, using this representation he has found also on expansion of (1) in powers of $\varepsilon = \ln \frac{1}{\rho}$ ($0 < \varepsilon < \pi$ if $r = 1, 3$ and $0 < \varepsilon < \frac{\pi}{2}$ if $r = 2$) with coefficients which are expressed by some integrals. For all $r \in N$ and $\rho \to 1-0$ an asymptotic expansion of (1) in powers of $\varepsilon = \ln \frac{1}{\rho}$ was found in the paper of Yu.I.Harkevich and I.V.Kalchuk in 2004. Note that the coefficients of the their expansion are expressed explicitly only for the first r terms and the remaining ones are defined by a complicated recurrence formula.

We would like to expose, for some more general series that (1), an expansion in powers of $\varepsilon = \ln \frac{1}{\rho}$ having explicit coefficients and, as a corollary, an expansion in powers of $(1-\rho)$ also with explicit coefficients.

ПРИБЛИЖЕНИЕ ПЕРИОДИЧЕСКИХ ФУНКЦИЙ В РАВНОМЕРНОЙ МЕТРИКЕ ПОЛИНОМАМИ ТИПА ДЖЕКСОНА

В. В. Жук
Санкт-Петербургский государственный университет
zhuk@math.spbu.ru

Пусть

$$\Phi_n(t) = \frac{1}{2\pi(n+1)} \left(\frac{\sin \frac{(n+1)t}{2}}{\sin \frac{t}{2}} \right)^2$$

— ядро Фейера, C - пространство непрерывных 2π-периодических функций f с нормой $\|f\| = \max_{x\in\mathbb{R}} |f(x)|$, $t_k = \frac{2\pi k}{n+1}$,

$$J_n(f, x) = \frac{2\pi}{n+1} \sum_{k=0}^{n} f(t_k)\Phi_n(x - t_k)$$

— полиномы Джексона функции f,

$$\sigma_n(f, x) = \int_{-\pi}^{\pi} f(x+t)\Phi_n(t)dt$$

— суммы Фейера функции f, $\omega_r(g,h)$ - модуль непрерывности порядка r функции g, $\widetilde{F}$ - функция, тригонометрически сопряженная с первообразной для функции $f-\frac{1}{2\pi}\int_{-\pi}^{\pi} f$.

Полиномы $J_n(f)$ обладают (см. [1, с. 35,36]) свойствами: $J_n(f)$ — тригонометрический полином порядка не выше n, $J_n(f,t_k)=f(t_k)$, $J_n'(f,t_k)=0$. Представляет интерес вопрос: как связано поведение величины $\|f-J_n(f)\|$ со структурными свойствами функции f. Он изучался в ряде работ различных авторов (см., например, [2;3] и указанную там литературу). В частности, в [3] показано, что для любой $f\in C$

$$\frac{1}{n+1}\max_{0\le k\le n}(k+1)\|J_n(f)-f\|\asymp\alpha\left(f,\frac{1}{n+1}\right),$$

где символ «$\asymp$» не зависит от n и f,

$$\alpha(f,\delta)=\delta\left\|\int\limits_{\delta}^{\pi}\frac{f(\cdot+t)+f(\cdot-t)-2f(\cdot)}{t^2}dt\right\|+\omega_1(f,\delta).$$

Мы устанавливаем результаты следующего типа:

$$E_n(f)+\|J_{4n-1}(f)-f\|\asymp\omega_1\left(f,\frac{1}{n+1}\right)+(n+1)\omega_2\left(\widetilde{F},\frac{1}{n+1}\right),$$

$$\sup_{\alpha\in\mathbb{R}}\|J_n(f(\cdot+\alpha))-f(\cdot+\alpha)\|\asymp\omega_1\left(f,\frac{1}{n+1}\right)+(n+1)\omega_2\left(\widetilde{F},\frac{1}{n+1}\right).$$

При этом серьёзное внимание уделяется постоянным, входящим в полученные неравенства. В качестве примеров приведём следующие утверждения:

$$\|f-J_n(f)\|\le(2\pi^2+3)\omega_1\left(f,\frac{1}{n+1}\right)+2(n+1)\omega_2\left(\widetilde{F},\frac{1}{n+1}\right),$$

$$\|J_n(f)-\sigma_n(f)\|\le(2\pi^2+3)\omega_1\left(f,\frac{1}{n+1}\right)+2(n+1)\omega_2\left(\widetilde{F},\frac{1}{n+1}\right).$$

В основе приведённых выше соотношений лежат теоремы общего характера, применимые и к другим методам приближения. Отметим, что подходы, использованные в обсуждаемых исследованиях, в значительной степени уже применялись автором ранее. В определённой степени они нашли своё отражение в монографии [4].

Литература

1. Зигмунд А. Тригонометрические ряды // Т.2, М., 1965.

2. Szabados J. On the convergence and saturation problem of Jackson polynomials // Acta Math. Hungar. 24, No. 3-4 (1973), 399-406.

3. Xie T. F., Zhou X. L. The Jackson interpolation operator and construction of functions // Acta Math. Hungar. 112, No. 3 (2006), 237-247.

4. Жук В. В. Аппроксимация периодических функций // Л., 1982.

Index

Askari Hemmat A., 3
Averbuch Amir Z., 4, 5

Babenko V.F., 5, 6
Babenko Yu.V., 5, 6
Berezhnoi Eugeny, 7

Cattani Carlo, 8
Chernykh N.I., 10, 11
Chuanga Laurence Zsu-Hsin, 64
Chui Charles, 12

Dem'yanovich Yuri K., 13
Dodonov N.Yu., 15
Doong Dong-Jiing, 64

Evdokimov Sergei, 16

Farkov Yu.A., 17
Freeden Willi, 18

Gabr Mohamed Waleed C. O., 19
Garkovskaya S. A., 20
Ghaffari Ali, 22
Gritsutenko S., 23
Guttmann Moshe, 5

Han Bin, 21
Hosseini Rooteh, 22

Ilyasov Maxim, 23
Irodova Irina, 24

Kao Chia Chuen, 64
Khryashchev V.V., 25
Kim Yeonhyang, 26
King Emily, 27
Knyazeva T.N., 28
Kofanov V., 30
Kolomoitsev Yurii S., 31
Kosloff Dan D., 5
Kozlov I., 48
Kozyrev S.V., 32
Krasilnikova M.S., 34
Krishtal Ilya A., 35
Krivoshein Alexander, 35
Kumar Kaushik, 37
Kumar Varinder, 37

Lawton W., 37
Lebedeva E.A., 38
Leviatan D., 40
Lukomskii S.F., 40

Makarov A.A., 41
Malozemov Vasily N., 42
Massopust Peter, 44
Miroshnichenko Valery, 45
Mishchenko E.V., 46

Novikov S.Ya., 46

Olevskii A.M., 47
Oreshko N.I., 28

Parfinovych N.V., 5, 6
Peleshenko B.I., 47
Petukhov A., 48

Pevnyi Alexander B., 42
Priorov A., 48
Prosekov Oleg V., 42
Protasov V., 49

Radzilevskaya E.I., 50
Reinov Oleg, 52, 53
Ron Amos, 26
Rozhenko Alexander I., 54

Severov Pavel G., 55
Shelkovich V.M., 56
Skopina M., 57
Skorokhodov D.S., 5, 6
Solovjova Natalia A., 57
Stasyuk S.A., 58
Subbotin Yu.N., 10, 11

Tchobanou Mikhail K., 59
Terekhin P. A., 60

Unser Michael, 61

Volkov Yuri, 62
Volokhov V., 48

Wronicz Zygmunt, 63
Wu Li-Chung, 64

Yazdani Fard Z., 3
Yazdani Azad, 65
Yunakovsky A.D., 66

Zakharov Victor G., 67
Zaramensky D.A., 25
Zastavnyi V.P., 68
Zheludev Valery A., 4, 5
Zhuk V. V., 69